

**SELF INSTRUCTIONAL MATERIALS IN MATHEMATICS**  
**FOR**  
**PRIMARY SCHOOL TEACHER**

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## PREFACE

Primary School Mathematics Curriculum has been revised based on the National Curriculum Framework 2000 at the National level. Consequently, new textbooks have been developed by the NCERT. In view of this, it is felt that primary school teachers should be provided with some extra inputs to help them in transacting the new mathematics curriculum effectively in the classrooms. It has also been often observed that the students' performance in Mathematics at secondary and higher secondary levels has not been very satisfactory because of the lack of proper development of basic Mathematical Concepts at the Primary School level. Primary school stage is the most important stage in the development of basic concepts and hence the primary teachers have the added responsibility to see that these basic concepts are developed in each and every child and the teachers are to be helped in this endeavour. As intensive inservice raining covering all the teachers in all these aspects may not be practical, it is thought that help may be given to the teachers in the form of self-instructional materials, which the teachers can study on their own whenever they find time.

The specific objectives of this programme are

1. To identify topics in the primary mathematics curriculum that are found to be difficult by the teachers to transact in the classroom.
2. To develop self-instructional materials on the identified topics.

For the task of identification of difficult topics, a questionnaire was developed by the RIE faculty. A team from RIE visited various KVS schools in the four Southern States to interact and collect the responses of

the practicing primary teachers to this questionnaire. The responses were analyzed and the topics for development of self-instructional materials were identified.

Self-Instructional Materials were developed in a 10-day workshop involving Mathematics and Mathematics Education experts as well as some practicing Primary School Mathematics teachers.

The developed materials were discussed with the primary school teachers of various KVS schools of the four southern states and their opinions were sought by the RIE team. Finally, the materials were reviewed in a Workshop inviting Mathematics and Mathematics Education Experts. Based on the review, the self-instructional materials have been finalized in the present form. The material is now ready for dissemination.

We are indeed grateful to Prof J S Rajput, Director, NCERT for all his encouragement to undertake this programme. We also thank the experts, primary school teachers from DMS, KVS and other schools, school authorities, RIE mathematics faculty members, members from DEE, CPU and to everyone who has helped in the development of the materials.

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## **NUMBER, NUMERATION AND OPERATION ON NUMBERS**

Numbers are the very first concept a child encounters in Mathematics. Although numbers are quite abstract, due to the repeated exposure and familiarity, they appear to be quite concrete. If the teachers at the very beginning take enough care to ensure that the concept of numbers, and operations on them are properly set in the minds of young kids, there will be no problem in developing further concepts.

Often the mere ability to read and write numbers by the kids is considered sufficient by the teachers. This is not a healthy situation. There is one more thing which is usually forgotten or ignored, that is, the feel of numbers, which further leads to the appreciation of the beauty of numbers and their properties. This can be easily achieved by creating situations to perform simple activities which are not only educative but also joyful. The moment children learn to enjoy mathematics, the battle is almost won. But this is possible only when the teachers, especially primary teachers take extra pains to show devotional attitude, give affectionate treatment to kids, try to promote loud thinking amongst children, be patient enough to listen to their queries, inspire them to probe and explore, be a facilitator rather than a dictator, be a co-operator rather than a manager or just a hard task master.

In the present write up we have made an effort to suggest to the teachers to incorporate the points mentioned above. Over 50 activities are suggested which require no expensive items and are also easy to perform in the classroom or even sometimes outside. Mostly they are joyful, every care is taken that children are inspired not scared. But the real success would totally rest on how the teachers interact with the children because more than the tool, the performer is important.

Especially it is desired that the teachers exposed to the suggested activities should design their own new activities and experiment with them. Otherwise the very purpose of this work will be defeated.

## CARDINAL AND ORDINAL NUMBERS

Every time we go out to buy our grocery or shopping, we speak to the shopkeeper to give us (for example) a kilogram of rice or three bars of soap or ten chocolates. Here we quantify the items without having in-depth knowledge about cardinal numbers. Similarly when someone asks us for directions to reach a particular place, we may say "Take the first left turn, then move straight ahead to reach the place". Yet again we have used the mathematical concept of ordinality to specify the location of the place without actually being aware of it. Thus we see that these two concepts of cardinal and ordinal numbers are very frequently used in our daily lives. When speaking about children, we see that even before they enter into schools, they have some crude notion about numbers. They group objects according to their size or colour and gradually start counting. They match an item in one group to an item of another group. These two processes of matching and counting are the processes upon which our simplest ideas of cardinal and ordinal numbers are based.

### Cardinal Numbers

Cardinal numbers are always associated with a set. They represent the size of a set of objects. Cardinal numbers usually answer the question of the type "How many?"

Example 1 : The teacher asks the students

How many children are present in the class today ?

The students give it to be 29. (Say) Here again 29 represents the size of the collection (here it is the collection of students). So 29 is said to be a cardinal number.

Example 2 How many flowers are there in the box ?



The reply to this question is 7. Here the number 7 represents the size of the collection. So we call 7 to be a cardinal number.

**Example 3 :** The teacher places a box containing certain assorted objects like stones/chalks/ flowers/ twigs, etc in front of the class. She then asks a student to pick at random from the box. The remaining students are asked to count the number of objects picked by the child. This process is continued to strengthen the skill of counting in the students.

- ☞ A number which represents the size of a collection is called a cardinal number.
- ☞ Cardinal numbers are quantitative.

**Activity 1 :**

Take 4 books pertaining to different subjects say English, Maths, Science, Social Studies. Arrange these books in the following order.

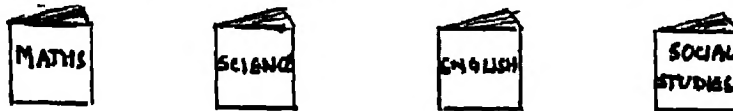
Case (i)



Ask the students: How many books have been arranged here ?

The students count and reply 4.

Case (ii) Now rearrange the books in the following way.



Tr . How many books are arranged ?

St. 4

Case (iii) Change the arrangements of the books to :



Tr How many books are arranged ?

St 4

Consider few more such rearrangements. In each case we see that the number of books which have been arranged remains the same irrespective of the arrangement of the books (i.e. the position of the items has not been taken into consideration).

Here the number 4 stands for the number of books, so it is a cardinal number. Thus we see that the size of a collection remains the same regardless of the sequencing of the items. [Sequencing fixes the position of the articles]

So we say that

- Cardinal numbers are non-positional numbers. They are quantitative in nature

Activity 2 : The teacher can prepare flash cards bearing numbers which are to be matched by the student to a corresponding collection of assorted items.

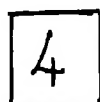
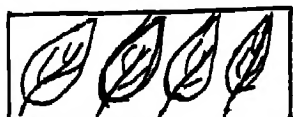


Step 1 : Few number cards are to be prepared by the teacher, like the ones shown above and places all of them together.

Step 2 : Teacher then collects few items and make various groups containing different number of items.



Step 3 : The teacher then instructs the students to count the items in each group and place a number card which represents the number of items besides the group.





This enables the teacher to check if the child can

- count the objects given to him.
- Identify the number card which represents the size of the set.
- Match the size of the collection with its corresponding number card.

### Activity 3 :

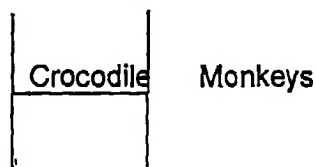
The teacher groups the students in the class into smaller clusters of different sizes (i.e. the number of students in each of the group differs from one another).

Select a student and assign him a number card bearing some number. The child is then asked to find the group which contains the same number of students as represented by the card. Every correct action by the student earns him 5 points. Repeat the activity by changing the sizes of the group. The student with the highest number of points is declared the winner.

Activity 4 : Let us consider the game "The monkey and the crocodile". This can be an outdoor game.

Step 1 : Choose a student in the class to be the crocodile. Remaining students are monkeys then.

Step 2 : Then mark an area of the ground by drawing 2 lines (parallel to one another). This is the region which the crocodile must guard



Step 3 : The crocodile is asked to stand in this region and guard his region against the monkeys. He must prevent the others from crossing over to the safer regions. (The crocodile can catch the monkey only within this area).

Step 4 : The game begins with the player acting as the crocodile calling out a number. The other children who act as the monkeys (not a very large one) are asked to collect as many objects as called out by the crocodile in order to

safely cross the crocodile's region, failing which they become the next crocodile.

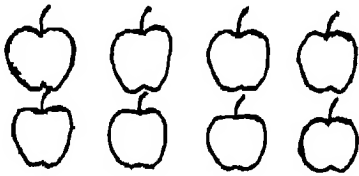
Step 5 : Once all the monkeys have cross over to the other area, they are to come back to their earlier bank, by running across the crocodiles region again, careful enough not to be caught by him. If caught, they become the next crocodile. This game can be continued for as long as possible.

Activity 5 : Instruct the students to colour the given number of items from a collection.

Example : Colour the following :

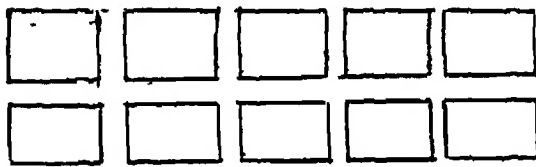
(a)

5 apples



(b)

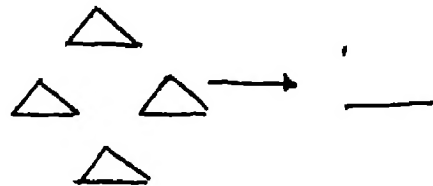
7 bricks



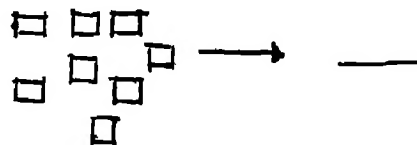
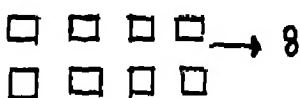
Activity 6 : Set up 2 collections of objects consisting of different number of items. Provide the child with the size of the first collection and ask the child to find the size of the second collection.

Example :

(a)



(b)



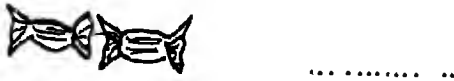
**Exercises :**

**I. Count and write the number :**

(a)



(b)



(c)



**II. Represent the following number pictorially.**

(a) 9

(b) 13

(c) 4

**III. Match the following**

(a) (i) 15

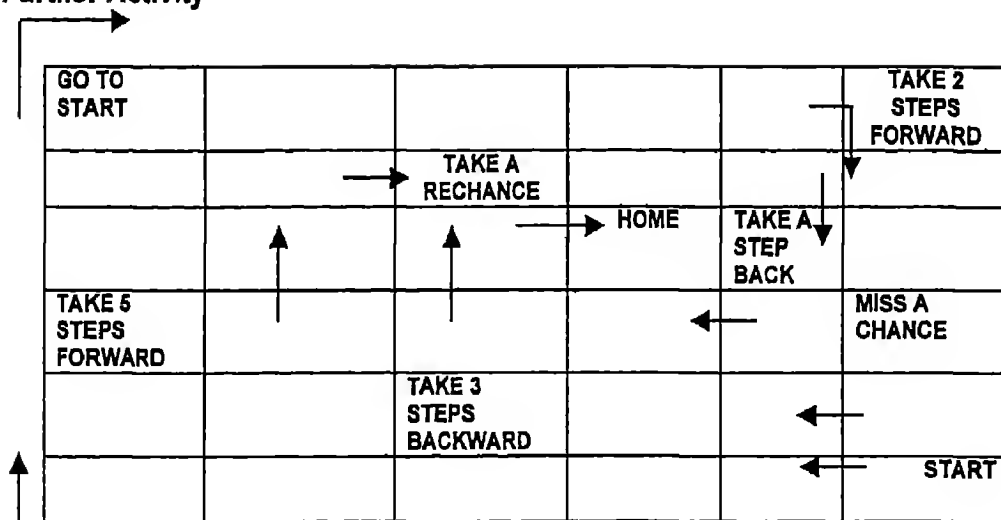
(b) (ii) 12

(c) (iii) 14

(d) (iv) 10

[Ans: (a)- (iv), (b) - (iii), (c) - (i), (d) - (ii) ].

### Further Activity



The coins required to play the game can be prepared by taking some paper pieces and shading them with different colours.

### How to play the game

1. Each player selects a coin of his choice with which he would play the game.
2. The game board as shown above has some blocks with certain instructions written. Whenever the player reaches that position, he should play as directed.
3. The game starts with throwing of a die and the player who gets 5 first starts the game.
4. The coins are moved as many places on the board as the number shown by the die
5. If the player gets a 1, then he is allowed to throw the die a second time. But if again on the second throw he gets 1 then he would lose a chance to play.
6. Moving through the board and reaching the "Home" block is the goal of the game. The player who reaches home first is declared as the winner.

## Ordinal Numbers

The word ordinal is derived from a Latin word meaning 'order', 'sl. No' or 'rank'. These numbers designate the position of an object in a series from a given reference point. After the first, second, third, from the fourth onwards the suffix 'th' is frequently attached to cardinal numbers.

i.e. 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, 5<sup>th</sup>, .....etc.

Example : Consider the following examples.

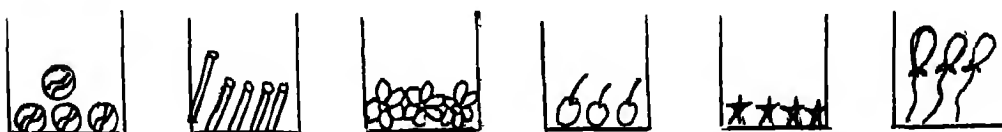
- The *fourth* shelf from the bottom of the cupboard contains the cookie jar
- The *tenth* boy from the right in the line is Rahul.
- The *fifth* bag from the back of the row belongs to me

In these three examples the terms fourth, tenth, fifth are used to show the position of certain objects from a given reference point (here – bottom, right, back).

These numbers which designate the placement of an item in a given series from a reference point are called *ordinal numbers*

[Note The reference point must be specified to fix the position of the object.]

Example 2 . Consider the following arrangement of objects in 6 containers



Let us define the position of the container containing *apples* from two different referential points. From the *left* it stands at the 4<sup>th</sup> position while from *right* it stands at the 3<sup>rd</sup> position.

Similarly from the left, the first container has balls while from the right the first container has leaves. So it is clear that as the point of reference from which the object is considered changes, the position of the object in the series changes.

Thus, ***ordinals are positional numbers related to the reference point.***

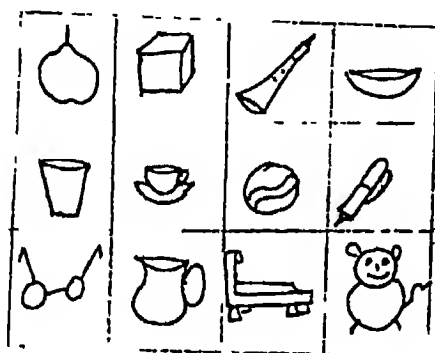
**Activity 1 :** Collect a group photo showing the Indian cricket team (from any sports magazine/newspaper) and make as many photocopies of it as the number of students in the class. Distribute a copy of the picture to each student and carry out this activity

How to proceed ?

Call out the name of a player (familiar to all students) say Tendulkar and ask the student to define his position in the photo and encircle it

**Activity 2 :**

Given the position of an article, the object given in the series must be identified Eg. consider the following example where a few articles are placed in the following manner.



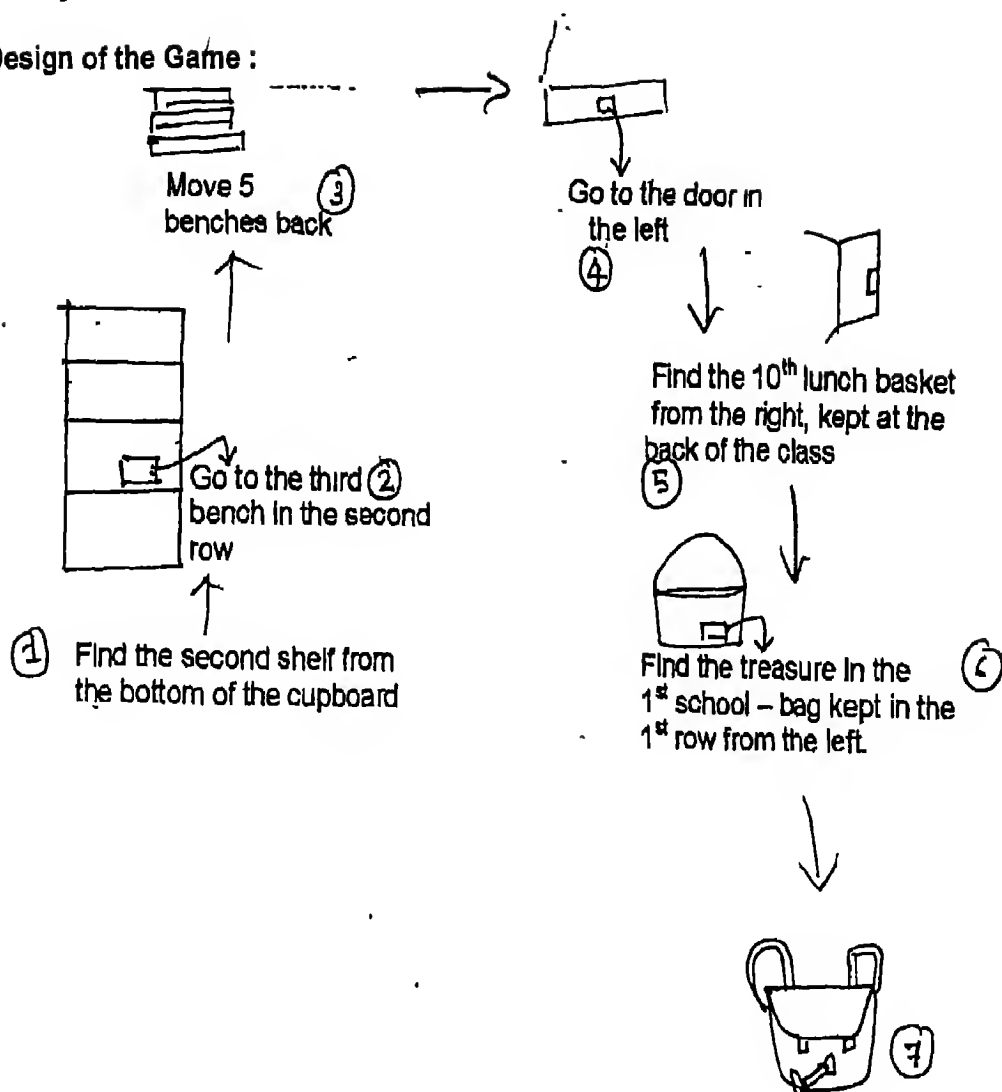
Instruction: Encircle the following articles in the group and name them.

- The third item from left in the second row from the top
- The first item from right in the first row from the bottom

These activities enable the child to locate the positions of the articles accurately

**Activity 3 :** This activity is called as "Treasure Hunt". Here the teacher places instructional cards at some strategic places in the classroom (like shelves of a cupboard, benches in the class, etc). The student moves from one instructional card to another by following the instructions given in the card. The aim of the game is to find the hidden treasure (To appreciate ordinal numbers).

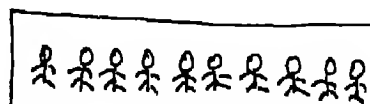
### Design of the Game :



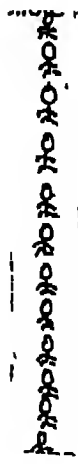
Such activities which emphasise the position of articles in a series can be further developed by the teacher keeping in mind the articles available in and around the class.

### Exercises :

1. In a group photo, ten children are seated. Hasib is the fourth boy from the right. Encircle his position in the picture.



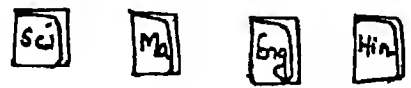
2. Encircle the child who is at the twelfth position from the bottom.



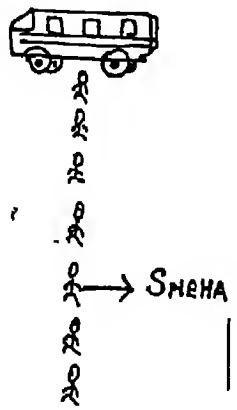
3. Colour the third apple from the left red and the third apple from the right green.



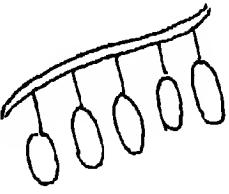
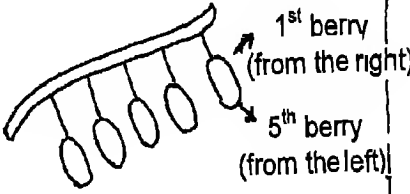
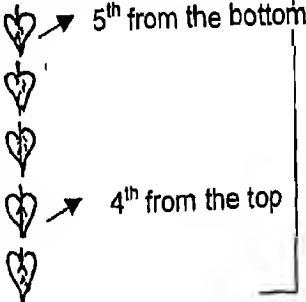
4. Designate the position of Maths book from the right and left



5. Seven people are standing in a queue to get into a bus. Sneha is one of them. What is her position in the queue?





<p>4 No referential point is required to define the size of a set. Eg 5 as a cardinal number.</p>  <p>Here 5 shows the number of berries on the branch</p>	<p>4. Reference point is very necessary to define the position of the object.</p>  <p>Here 5 is depicting the position of the given item from a reference point</p> 
<p>Number of leaves = 4 (Draw the picture)</p>	

**Further Activity :** Aim of the activity is to use the concept of cardinal and ordinal numbers simultaneously

How to conduct the activity

Step 1: Place a few chairs in a row as shown in the figure

Step 2 Collect a few items and make them into groups containing different number of items. Assign a group each to every child in the class

Step 3 . Ask the students to count the number of items in the group assigned to them.

Step 4 The student is required to count the number of items, find the size of the group given to him and occupy the seat which is placed at that position number as the number of items counted by him.

[Note: Here the teacher must specify the referential point ]

Number of items = 3

Referential direction = left to right



## THE PLACE VALUE

### Introduction

Man's first recognition of the need for numbers probably arose from his need to count, which resulted in the invention of numbers. Each number was given number name. There was no problem so long as only the number names were just spoken.

But in due course of time, the need for writing numbers was felt. For a long time the numbers one, two, three, .. were represented by bars like |, ||, |||, ...

In the beginning it did not bring any difficulty because in those days nobody felt the need for large numbers in their practical life. But as population increased and civilization grew, the need for large numbers was felt. The difficulty arose in the prevailing way of representing numbers by bars. This system was no longer easy to represent a large number, because by mere glance of a heap of bars one could not get the feel for the quantity of the bars present in the heap. So a need for a better scheme of representing numbers was felt.

There was considerable variety in the systems of numeration used to form large numbers.

Attic system of numeration was used in ancient Greece. Slavic numeration was used by the Slavic people of the South and east of Europe. Positional system of numeration of Babylonian, Roman numeration of the ancient Romans, Decimal system of numeration by the ancient Indians are some of the systems of numeration used by the ancient people.

Decimal system of numeration used by the ancient Indian was spread to other parts of the world and is today the generally accepted system of numeration.

**Explanation**

The decimal numeration system is one of the truly great invention of the human kind. In the Decimal system of numeration we can represent a number, however large it may be by using only 10 symbols namely 0,1,2,3,4,5,6,7,8 and 9. These symbols are called digits.

The value of any number is the sum of the values that the digits contribute by the virtue of their position/ place in the number expression. In fact, the number expression is based upon the conception of place value.

The main ideas that underlie in the operation of the Decimal numeration system are as follows.

**Idea of a Constant Value or Face Value**

Each digit of the 10 digits i.e. 0,1,2,3,.. 9 represents constant value which is called face value of the digit. The face value of the digit is determined by its place within the series of the digits.

Example: In any position within the numeral, a four has a value that is always one more than three, one less than five and so forth i.e. the face value of 4 is 4, the face value of 8 is 8.

**Idea of a place value**

Each digit used in a numeral represents the place value based upon its place within the numeral.

The one's place is the reference position in which grouping of less than 10 ones are expressed. In right to left order, each place has a value 10 times larger than the immediate right. Hence the place values are the successive powers of ten, as listed below.

Place Name	Place Value	Powers of 10
Units	1	$10^0$
Tens	10	$10^1$
Hundreds	100	$10^2$
Thousands	1000	$10^3$ and so on

1000	100	10	1	
Thou- sands	Hund- reds	Tens	Ones	<p>One's place is the basic reference position. A digit in this place represents its face value only.</p>
				<p>Tens place: A digit in this place indicates the number of groups of 10. So, the digits in this place represents its place value 10 times as large as its face value.</p>
				<p>Hundredth place. A digit in this place indicates the number of groups of 100. So, the place value of a digit is 100 times its face value</p>
				<p>Thousandth place A digit placed in this position depicts a number of groups having a thousand numbers each. So the place value of a digit is 1000 times its face value.</p>

### Activity 1 :

Keep a pile of sixty seven 1 rupee coins on the table. Call a group of children (maximum – 5). Ask them to count the coins individually. As the tenth coin is counted, ask them

T How many rupees do you have ?

S 10 rupees

Teacher can show one 10 rupee note and ask him

T Can you exchange this one 10 rupee note for ten 1 rupee coins ?

S Yes

Now children can be made to replace the ten 1 rupee coins for one 10 rupee note in exchange. The next ten 1 rupee coins can be replaced by another 10 rupee notes.

Let them continue the process until the piles of coins come to an end

At the end of the process, they find

Six – 10 rupee notes and seven 1 rupee coins on the table

Now the teacher can ask the class to observe coins which are counted by their friends and ask them.

T: How many 1 rupee coins can make one 10 rupee note ?

S: Ten 1 rupee coins can make one 10 rupee note Now teacher can infer the idea **10 ones make 1 ten, 10 ones one 10.**

T: How many 10 rupee notes are there on the table ?

S: Six – 10 rupee notes

T: What is 6 tens mean ?

S: 6 tens mean 60

T: How many 1 rupee coins are there on the table ?

S: Seven 1 rupee coins

T: What is 7 ones mean ?

S: 7 ones mean 7.

T: Add and find the value of 6 tens and 7 ones

S: 6 tens 7 ones make 67.

Teacher can highlight it on the blackboard

6 tens 7 ones make 67

i.e.  $60 + 7 = 67$

With the help of the activity mentioned above, teacher can elicit the value of the examples given below

8 tens, 2 ones make	82
7 tens, 7 ones make	77
6 tens, 4 ones make	64
4 tens, 6 ones make	46
9 tens, 1 one make	91
1 ten, 9 ones make	19
5 tens, 0 ones make	50

Now teacher can infer the value of any number is the sum of the place values of its digits

**Activity 2 :**

Keep a pile of small sticks / match sticks on the table. Call a child and tell him to count loudly all the sticks and find out the number of sticks in the pile.

Let the number of sticks in the pile be fifty four.

Now a group of children (max – 5) can be called and told to count the sticks. As the tenth stick is counted, tell them to bundle it with rubber band. After 5 bundles they could not make any more bundles. Now teacher can ask them to separate the bundles from unbundled match sticks and group them in order.

T: How many bundles are there ?

S: 5 bundles

T: How many match sticks are there in 5 bundles ?

S: 5 tens or 50 match sticks

T: How many unbundled match sticks are there ?

S: 4 ones or 4

T: How many tens and ones are there in 54?

S: 5 tens 4 ones

Now teacher can infer

**54 means 5 tens 4 ones i.e.  $54 = 50 + 4$ .**

**Activity 3**

The similar kind of activity can be conducted with 45 match sticks and children can find out

45 means 4 tens 5 ones

$45 = 40 + 5$

Now teacher can ask the children to observe the number 54 and 45 and ask them

- T: Do 54 and 45 have the same digits in them ?
- S: Yes
- T: Name those digits
- S: Four and five
- T: What is the place value of 5 in 54 and 45 ?
- S: The place value of 5 in 54 is 50 The place value of 5 in 45 is 5.
- T: What is the place value of 4 in 54 and 45 ?
- S: The place value of 4 in 54 is 4 The place value of 4 in 45 is 40
- T: Why do the same digits in different positions differ in their place value?
- S: They differ in their values because they are placed in different positions

Now the teacher can infer that the value of the digit in a given numeral depends upon its positioning or placing.

Activity 4 Teacher can divide the class into 10 groups A pile of match sticks can be given to each group. As per the instruction given in the activity 2, the children can be asked to find the place value of the digits in the numbers given to them. Finally teacher can prepare a table and enter the finding outs of the children in it

Group	No. of match sticks	Place Value	
Group I	23	2 tens or 20	3 ones or 3
Group II	40	4 tens or 40	0 ones or 0
Group X	15	1 ten or 10	5 ones or 5

This chart can be displayed in the classroom.

**Suggested Activity:** Teacher can ask each child to draw the abacus for any 5 numbers from 1 to 100 On the chart paper, teacher can collect the charts and display them in the class Teacher can also insist the children to invite their friends in the other sections to witness their project work.

### Exercises

Write the numbers from 1 to 100 in order. By observing those numbers carefully ask the children to answer the following questions.

1 Find out all the numbers in which the digit 2 is in one's place.

Ans: 2, 12, 22, 32, 42, 52, 62, 72, 82, 92.

2. Find out all the numbers in which the digit 9 is in ten's place

90, 91, 92, 93, 94, 95, 96, 97, 98 and 99.

3 Find out all the numbers which have zero in one's place.

10, 20, 30, 40, 50, 60, 70, 80, 90, 100

4. Find out all the numbers in which the digit 6 has the place value 6 tens or 60

60, 61, 62, 63, 64, 65, 66, 67, 68 and 69

5. Find out all the numbers in which the digit 7 has the place value 7 ones or 7

7, 17, 27, 37, 47, 57, 67, 77, 87 and 97.

6. In which number the digit 3 has the place values 30 and 3

Ans 33.

7 Which number do you get if the digit 5 is in ten's place and the digit 8 is in one's place ?

Ans 58

8 In a number 0 is in one's position and 9 is in ten's position Find out the number

Ans' 90

9. In the number 49, which digit is placed in ten's place? What is its place value ?

Ans' 4 is ten's place and its place value is 40.

10. Write the number, if the place values of the digits in the numbers are 8 and 60

Ans' 68

11 In the number 91, which one of the digit has the place value 1? Why does it have place value 1?

Ans' The digit 1 has the place value 1 because it is placed in one's place

12 In the number 99, which one of 9 has the place value 90 Encircle it

Ans 99



13. Among the number 58 and 85. In which number, the place value of 8 is 80 ? Give reasons.

Ans: 85. In 85, the digit is placed in ten's place

14. Four numbers are given below. In which number the digit 3 does not have the place value 30. Underline it. Give reason for your answer.  
a) 30              b) 13              c) 31              d) 33

Ans: 13, only in 13, the three is placed in one's place and its place value is 3.

By making suitable/necessary modification activity 1, 2, 3 and 4 can be conducted to find out the place value of the digits in the 3-digit number.

To enrich the understanding of the concept. Teacher can solve the following exercises.

#### Exercise 1 :

Teacher can ask the children to observe the numerals 456 and 645 and ask them some questions.

T: What do you observe about the digits used in these two numbers ?

S: The digits 4, 5 and 6 are used in both the numbers

T: What does 456 mean ?

S:  $456 = 400 + 50 + 6$

T: What does 645 mean ?

$645 = 600 + 40 + 5.$

T: Are these two numbers differ in their values ?

S: Yes, they differ in their values.

T: What causes the two numbers to differ in their values, although they involve the same digits ?

S: The place value of the digits causes the two numbers to differ.

#### Exercise 2

Write a number for  $400 + 40 + 4$ . Do the three 4s differ in their place value? If so, why ?

a)  $400 + 40 + 4 = 444$

b) Yes. Three 4s differ in their place value.

- c) Three 4s are placed in three different places namely hundreds, tens and ones. So they have different place values.

### Exercise 3 :

Write the number 772 in the expanded form. What are place values of two 7s? Why do they differ in their values ?

- a)  $772 = 700 + 70 + 2$   
 b) The place value of 7 in tens place is 70. The place value of 7 in hundreds place is 700.  
 c) They differ in their values because they are placed in different places of the given number.

### Idea of Place Value of 0

The digit zero in a particular place indicates that the place is vacant. Thus zero keeps the other digits in their proper places.

### Exercise

Read the number 107.

- T: What does the 7 stand for ?  
 S: 7 ones or 7.  
 T: What does the 1 stand for ?  
 S: 1 hundreds or 100  
 T: What does the 0 stand for ?  
 S: 0 tens or 0  
 T: There are no tens. What number would you have if you leave out the zero in 107 ?  
 S: 17.  
 T: Can you read 17 as one hundred seven?  
 S: No, we can read it as seventeen.

Now teacher can infer Though 0 does not have any place value which ever the position it holds. But it helps to keep the other digit in their proper positions.

**Exercise 2 .** Ravi thought that, there is no ones in 120 rupees. There is no reason to write the zero. So he wrote Rs.12. State whether he is right or wrong. Give reason.

Ravi was wrong. He writes Rs.12, it means 12 rupees and not 120 rupees

### Exercises

1 Find out the number which has digit 6 in one's place, 7 in ten's place and 3 in hundreds place

Ans: 376.

2 What number do you get, if the digit 1 is in hundreds place, 9 in tens place and 8 in ones place ?

Ans: 198

3. A number has the digits 5 in tens place, 2 in ones place and 6 in hundreds place. What is that number ?

Ans. 652

4 The place values of three different digits in a 3-digit number are 7, 80 and 200. Find out the number

Ans:  $200 + 80 + 7 = 287$

5 In which 3-digit number, the digit 7 has the place values 7, 70 and 700

Ans: 777

6 In a 3-digit number the digit has the place value 600. Digit 1 has the value 10 and the digit 9 has the value 9. What is that number ?

Ans. 619

7 Write the 3-digit number, in which the place values of 5 are 500 and 5 and the place value of 2 is 20

Ans: 525

8 Find out all the 3 digit numbers in which the place value of the digit 6 are 600 and 60

660, 661, 662, 663, 664, 665, 666, 667, 668, 669

9 Find out all the 3 digit numbers in which the place values of 9 are 900 and 9

909, 919, 929, 939, 949, 959, 969, 979, 989, 999

10. Find out all the 3 digit numbers in which the place value of 1 is 100 and 10

Ans. 110, 111, 112, 113, 114, 115, 116, 117, 118, 119

11. Teacher asked Raju to write the numeral for five hundred seven. He wrote 57. Was he correct ? If not, write the correct numeral

Ans. Raju is wrong. b) 507.

12. Lalitha wrote the number name for 707 as seven hundred seventy. Was she correct ? If not, write the correct number name

Ans. She is wrong. Seven hundred seven

13. Ravi wrote seven hundred seventy as 70070. Was she correct? If she is not correct, why? She was not correct

i.e.  $700 + 70 = 770$

The sum of the place value of the digits give the value of the number

14. Lalitha said that the number 600 means  $6 \times 100 + 0 \times 10 + 0 \times 1$ . She came to the conclusion that there are no tens and no ones and she asked why do we need 0s in 600? Can you answer ?

Ans. If we did not write 0 tens and 0 ones, the number will become 6 instead of 600

By the previous knowledge about the place value of the digits in a 2-digit and 3-digit number, the teacher can help the children to find out the place value of a digit in a 4-digit, 5-digit numbers

Using abacus and placing the digits in the place value chart are the best ways of finding out the place value of a digit in a large number.

### Exercise

If you change the place of 6 and 7 in the number 6723.

- Will the number you get be larger or smaller than 6723 ?
- Change the place of 6 and 2

Is the number you get larger or smaller than 6723 ? Explain

- T. Which number will you get if you change the place of 6 and 7 in 6723?  
 S 7623  
 T Is 7623 smaller or larger than 6723 ?  
 S Larger than 6723.  
 T: Give reason or explain.  
 S; 6723 and 7623 are 4-digit numbers The place value of 6 in 6723 is 6000 The place value of 7 in 7623 is 7000 7000 is greater than 6000. So 7623 is greater than 6723  
 T Which number will you get if you change the place of 6 and 2 ?  
 S: 2763  
 T Is 2763 smaller or larger than 6723 ?  
 S: Smaller than 6723.  
 T: Explain  
 S The place value of 2 in 2763 is smaller than the place value of 6 in 6723 So 2763 is smaller than 6723.

### Exercises

- 1 In the number 12880 (a) which 8 represents the greater value? (b) How many times as greater ?  
 Ans: (a) In 12880, 8s are placed in hundred's place and ten's place. So 8 in hundred's place is greater than 8 in ten's place i.e 800 is greater than 80 (b) 800 is 10 times as 80 So 8 in hundred's place is 10 times 8 in ten's place
2. State whether the following statements are true or false If it is false, write true statement
  - a) The place value of 4 in 64356 is thousands  
 Ans False The place value of 4 in 64356 is 4000
  - b) The place value of 8 in 83271 is 80000  
 Ans True
  - c) The place value of 0 in 75023 is 100  
 Ans: False The place value of 0 in 75023 is 100
  - d) The place value of 1 in 91656 is 100  
 Ans False The place value of 1 in 91656 is 1000.

e) The place value of 3 in 28531 is tens.

Ans: False. The place value of 3 in 3 is 30.

f) In a 4-digit number only the digits 5 and 0 are used. If the place values of 5 is 5000, 50 and 5 in which place 0 should be placed and what is the given number?

Ans: 0 should be placed in hundreds place The given number is

$$5000 + 50 + 5 = 5055.$$

## COMPARISON OF FRACTIONS

### Introduction

The basic needs of everyday life led to the introduction of common fractions like  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$ , etc and also comparison of these fractions. This material intends to provide a sample of desirable and appropriate teaching-learning strategies for comparing two fractions and then put more than two fractions in ascending or descending order. In this section, we define the relation "is greater than", "is equal to" and "is smaller than" on fractions and then show that it is possible to order fractions in a manner similar to the ordering of the whole numbers

The teachers are expected to read this material and use it in the classroom as functional modules. The basic concern of this material is to help the teacher to understand how to enrich/reorient their methodology of teaching so as to help all learners achieve satisfactory standard/mastery level of learning. Efforts are made to integrate evaluation with the teaching-learning activities. The teacher should ensure that the following competencies have already been developed in the children before asking them to compare two fractions.

- A fraction is a certain number of equal parts of something (a WHOLE)
- The line between the numbers in a fraction means that the numerator is divided by the denominator
- There are two types of fractions – (i) proper fractions, (ii) improper fractions.
- 1 is greater than any proper fraction
- Any improper fraction is greater than 1, also greater than any proper fraction
- Any improper fraction can be converted into (changed into) a mixed number and vice-versa.
- Change an improper fraction to a mixed number and a mixed number to an improper fraction

- Change a fraction to an equal fraction with a different denominator or generate equivalent fractions of a given fraction.
- Find a common denominator of two/several fractions.

### **Skills/Competencies to be developed**

- COMPARE the given two fractions.
- ORDER the given fractions in ascending order/descending order

### **Sub-Competencies to be developed under comparison of two fractions:**

1. Compare two proper fractions with the same denominators
2. Compare two unit fractions.
3. Compare two proper fractions with the same numerators
4. Compare two proper fractions with different numerators and denominators.
5. Compare a whole number and a proper fraction
6. Compare a whole number and an improper fraction
7. Compare two improper fractions (greater than 1)
8. Compare an improper fraction (greater than 1) and a mixed number.
9. Compare two mixed numbers

### **Recall the following :**

**Proper fractions:** A fraction whose numerator is less than its denominator is called a proper fraction

Eg.  $\frac{1}{6}$ ,  $\frac{4}{15}$ ,  $\frac{7}{8}$  etc

**Improper fraction :** A fraction with its numerator equal or greater than the denominator is called an improper fraction.

Eg  $\frac{3}{2}$ ,  $\frac{7}{4}$ ,  $\frac{15}{15}$ , etc.

**Mixed Numbers :** Fractional numbers that composed of a whole number and a proper fraction is called a mixed number

Eg.  $2\frac{1}{4}$ ,  $1\frac{7}{10}$ ,  $6\frac{8}{9}$  etc.



To change an improper fraction to a mixed number we have to divide the numerator by the denominator,

$$\text{Eg. } \frac{17}{5} = 3\frac{2}{5} \quad \begin{array}{r} 3 \\ 5 \overline{)17} \\ \underline{15} \\ 2 \end{array}$$

To change a mixed number to improper fraction, multiply the whole number by the denominator of the fraction and then add the product to the numerator of the fraction

Eg. Convert  $2\frac{7}{8}$  into improper fraction .

$$2\frac{7}{8} = \frac{(2 \times 8) + 7}{8} = \frac{16 + 7}{8} = \frac{23}{8}$$

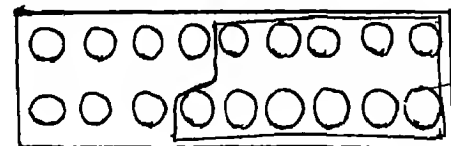
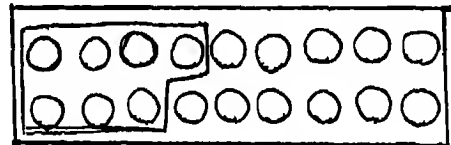
**To compare two fractions with the same denominator :**

Raju had eighteen 50 paise coins out of which he gave 11 coins to his daughter and 7 coins to his son Write each one's share in fraction form and compare them. Whose share is more ?

Son's share  $\frac{7}{18}$

Daughter's share  $\frac{11}{18}$

Now compare  $\frac{11}{18} > \frac{7}{18}$  since  $11 > 7$



Therefore, daughter's share is more This can be verified with a simple activity using 18 coins of 50 paise distributed among two students or using a picture as given below.

Daughter's share  $\frac{11}{18}$

Son's Share  $\frac{7}{18}$

$\frac{11}{18} > \frac{7}{18}$  since  $11 > 7$ .

We compare

Compare the two fractions of an inequality by comparing numerators if the denominators are same

For eg. which is greater  $\frac{5}{60}$  or  $\frac{17}{60}$

Since  $17 > 5$ ,  $\frac{17}{60} > \frac{5}{60}$ .

**Of two like fractions (the fractions with the same denominators) the one having the greater numerator is greater.**

Which is greater ?

i)  $\frac{7}{15}$  and  $\frac{12}{15}$

ii)  $\frac{29}{30}$  and  $\frac{14}{30}$

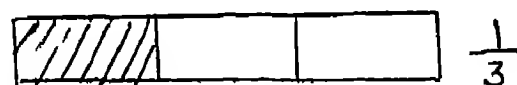
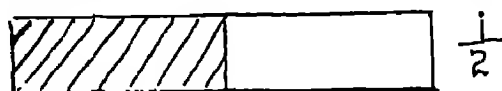
**To compare two unit fractions:**

Which is greater  $\frac{1}{2}$  or  $\frac{1}{3}$ ?

**Activity 1** Take two paper strips of same size. Fold one from the middle. Each part represents half. Take the other strip and fold it into three equal parts such that each part shows  $\frac{1}{3}$ . Keep one below the other as shown

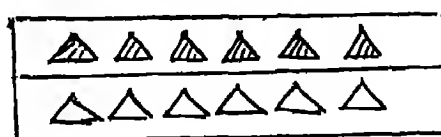
below

Here  $\frac{1}{2} > \frac{1}{3}$



Observe that  $\frac{1}{2}$  of a whole is greater than  $\frac{1}{3}$

**Activity 2 :** Take any collection having 12 items and consider it as a whole. Divide them into two equal parts.

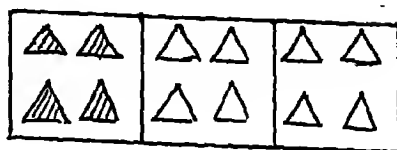


Observe that each part contains 6 and each part represents  $\frac{1}{2}$ .

Now divide the collection into three equal parts, each part representing  $\frac{1}{3}$ .

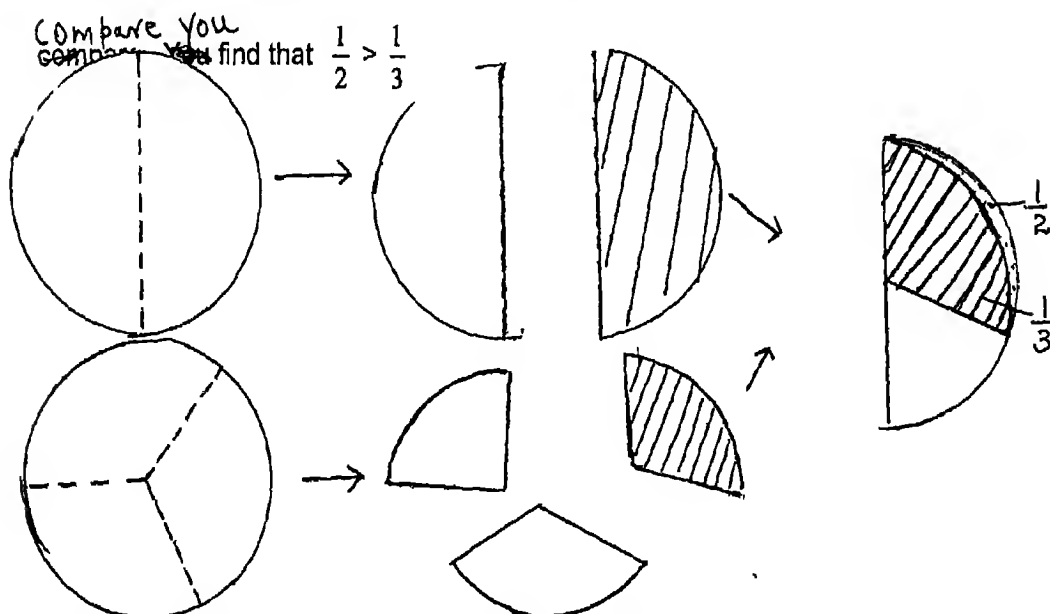
Observe that each part contains 4 items.

Hence  $\frac{1}{2}$  of 12 is 6 and  $\frac{1}{3}$  of 12 is 4.



$$\frac{1}{2} > \frac{1}{3}$$

**Activity 3 .** Make a *fraction disc*\* with cardboard. Use the sector pieces which represent  $\frac{1}{2}$  to one tenth to compare different fractions. Pick a piece which represents  $\frac{1}{2}$  and another piece which represents  $\frac{1}{3}$ . Observe and find out which is bigger. You may even keep one piece over the other and



How to make a fraction disc is given below. Make ten circular cut outs of same radius using cardboard / KG cardboard. Each circular cut outs represents a whole. Now cut one of circular card into two equal parts so that each part should represent  $\frac{1}{2}$ . Another circular card should be cut into 3

equal parts so as to represent one thirds. Now other cards into one fourths, one fifths, one sixths, one sevenths, one eighths, one ninths, one tenths. The remaining one to represent a whole.

$$(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55)$$

Thus there will be 55 pieces altogether from whole to one tenth of a whole.

Repeat the activity with the sectors representing  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$  etc. taking any two at a time. By the learning experiences gained, come to the conclusion that as the number of equal parts of a whole increases the fraction represented by the corresponding one part becomes smaller

For eg of the two unit fractions  $\frac{1}{5}$  and  $\frac{1}{4}$

$$\text{Since } 5 > 4, \quad \frac{1}{4} > \frac{1}{5}$$

*Of two unit fractions (the fractions with 1 in the numerator) the one having the smaller denominator is greater*

To compare two proper fractions with the same numerator.

**Method 1 · By comparing the denominators**

Of the two fractions  $\frac{3}{5}$  and  $\frac{3}{7}$ . Which is greater ?

$$\frac{3}{5} \text{ means } 3 \times \frac{1}{5}$$

$$\text{and } \frac{3}{7} \text{ means } 3 \times \frac{1}{7}.$$

$$\text{Since } \frac{1}{5} > \frac{1}{7}$$

$$\frac{3}{5} > \frac{3}{7}$$

*Of the two proper fractions having the same numerators and different denominators, the one having the smaller denominator is greater.*

For eg Which is greater  $\frac{15}{17}$  or  $\frac{15}{22}$ ?

17 is smaller than 22,

therefore,  $\frac{15}{17} > \frac{15}{22}$

**Method 2 : By equalizing the denominators :**

For eg, which is greater  $\frac{3}{5}$  or  $\frac{3}{7}$  ?

$$\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$$

$$\frac{3}{7} = \frac{3 \times 5}{7 \times 5} = \frac{15}{35}$$

Now compare,  $\frac{21}{35}$  and  $\frac{15}{35}$  since  $21 > 15$ ,  $\frac{21}{35} > \frac{15}{35}$  i.e  $\frac{3}{5} > \frac{3}{7}$ .

To compare two proper fractions with different numerator and denominators.

When a whole is divided into more number of equal parts and less number of parts are taken out of it the fraction which represents it becomes smaller.

**Method 3 . By the cross product :**

For eg which is greater  $\frac{3}{5}$  or  $\frac{3}{7}$  ?

$$\frac{3}{5} \quad \begin{array}{c} \nearrow \quad \nwarrow \\ \searrow \quad \nearrow \end{array} \quad \frac{3}{7}$$

$$3 \times 7 = 21 \text{ and } 5 \times 3 = 15$$

Now,  $21 > 15$ ,  $\therefore \frac{3}{5} > \frac{3}{7}$ .

**To compare two proper fractions with different numerators and denominators :**

Which is greater  $\frac{4}{9}$  or  $\frac{7}{8}$  ?

The fractions  $\frac{4}{9}$  and  $\frac{7}{8}$  neither have the same numerators nor the same denominators. In such cases we have to adopt one of the two methods (i) Equalising the denominator or (ii) the cross product.

**Method 1 : Equalising the denominator :**

$$\frac{4}{9} = \frac{4 \times 8}{9 \times 8} = \frac{32}{72}$$

$$\frac{7}{8} = \frac{7 \times 9}{8 \times 9} = \frac{63}{72}$$

Now compare  $\frac{32}{72}$  and  $\frac{63}{72}$

Since  $63 > 32$ ,  $\frac{63}{72} > \frac{32}{72}$

**Method 2 : The cross product method**

Consider the fractions.

$$\frac{4}{9} \text{ and } \frac{7}{8} \quad 4 \times 8 = 32; \quad 9 \times 7 = 63$$

$$63 > 32; \quad \frac{7}{8} > \frac{4}{9}$$

Case (i) When the numerator of the first fraction is less than the numerator of the second and the denominator of the first is greater than the denominator of the second, the second fraction is greater than the first.

For eg. In  $\frac{4}{12}$  and  $\frac{9}{10}$

$$4 < 9, \quad 12 > 10 \quad \text{Therefore, } \frac{9}{10} > \frac{4}{12}$$

Here  $\frac{4}{12}$  is nothing but  $4 \times \frac{1}{12}$  i.e. 4 pieces out of 12 pieces whereas

$$\frac{9}{10} = 9 \times \frac{1}{10} \text{ i.e. 9 pieces out of 10 pieces.}$$

Case (ii) When the numerator of the first fraction is greater than the numerator of the second and the denominator of the first fraction is less than the denominator of the second, then first fraction is greater.

For eg In  $\frac{8}{15}$  and  $\frac{5}{18}$ ,  $\frac{8}{15} > \frac{5}{18}$

Because  $8 > 5$  and  $18 > 15$

Or  $\frac{8}{15}$  is nothing but  $8 \times \frac{1}{15}$  and  $\frac{5}{18}$  is  $5 \times \frac{1}{18}$ .

Case (iii) Both the numerator and the denominator of the first fraction is greater than/less than the numerator and denominator of the second fraction.

For example, In  $\frac{4}{5}$  and  $\frac{7}{9}$  which is greater ? Also in  $\frac{11}{12}$  and  $\frac{5}{6}$  which is greater ?

In such cases it is necessary to use either equalizing the denominator method or cross product method

**Method 1 :**  $\frac{4}{5} = \frac{4 \times 9}{5 \times 9} = \frac{36}{45}$

$\frac{7}{9} = \frac{7 \times 5}{9 \times 5} = \frac{35}{45}$  since  $36 > 35$

$\frac{36}{45} > \frac{35}{45}$  or  $\frac{4}{5} > \frac{7}{9}$

**Method 2 :**

$\frac{11}{12}$        $\frac{5}{6}$        $11 \times 6 = 66; 12 \times 5 = 60$

Since  $66 > 60$ ,  $\frac{11}{12} > \frac{5}{6}$

The fraction  $\frac{a}{b}$  is greater than the fraction  $\frac{c}{d}$  if and only if the product  $a d$  is greater than the product  $b c$

$\frac{a}{b} > \frac{c}{d}$  iff  $\frac{a}{b} \nearrow \frac{c}{d}$ ,  $ad > bc$

To compare a whole number and a proper fraction:

Since all whole numbers are greater than part of one whole, comparison becomes easier.

For eg.,  $1 > \frac{1}{2}$  Here 1 represents  $\frac{2}{2}$ .

$2 > \frac{5}{7}$  Here 2 represents  $\frac{14}{7}$  when we equalize the

denominator.

$$\frac{2}{2} > \frac{1}{2} \Rightarrow 1 > \frac{1}{2}$$

$$\frac{14}{7} > \frac{5}{7} \Rightarrow 2 > \frac{5}{7}$$

To compare a whole number and an improper fraction.

Note

Every whole number is an improper fraction.

All improper fractions are not whole numbers.

Examples

1. Compare  $\frac{17}{17}$  and 3.

$$\frac{17}{17} = 1 \text{ since 3 is greater than 1, } 3 > \frac{17}{17}$$

2. Compare  $\frac{25}{4}$  and 5.

- i) Let us equalize the denominator

$$\frac{25}{4} = \frac{25}{4}$$

$$\frac{5}{1} = \frac{5 \times 4}{1 \times 4} = \frac{20}{4}$$

Now compare  $\frac{25}{4}$  and  $\frac{20}{4}$ .

Since  $25 > 20$ ,  $\frac{25}{4} > \frac{20}{4}$  or  $\frac{25}{4} > 5$



ii) Let us use the cross product method.

$$\begin{array}{ccc} \frac{25}{4} & \begin{array}{c} \nwarrow \nearrow \\ \nearrow \nwarrow \end{array} & \frac{5}{1} \end{array} \quad 25 \times 1 = 25 \text{ and } 4 \times 5 = 20$$

$$\text{Since } 25 > 20, \frac{25}{4} > \frac{5}{1} \text{ or } \frac{25}{4} > 5$$

**Comparison of improper fractions [other than whole numbers].**

Which is greater  $\frac{7}{2}$  or  $\frac{15}{8}$ ?

By equalizing the denominator we get

$$\frac{7}{2} = \frac{7 \times 4}{2 \times 4} = \frac{28}{8}$$

$$\frac{15}{8} = \frac{15}{8}. \text{ Now compare } \frac{28}{8} \text{ and } \frac{15}{8}$$

$$\text{Since } 28 > 15; \frac{28}{8} > \frac{15}{8} \text{ or } \frac{7}{2} > \frac{15}{8}.$$

By using the cross product method, we get,

$$\frac{7}{2} \quad \frac{15}{8} \quad 7 \times 8 = 56; 2 \times 15 = 30$$

$$\text{Since } 56 > 30, \frac{7}{2} > \frac{15}{8}.$$

*To compare two unlike fractions (fractions having different denominators), either they are changed into like fractions (by using the rule for getting equivalent fractions) and the numerators are compared or the cross products are found and the numerator as a multiplier which gives the greater number is the greater fraction.*

To compare an improper fraction and a mixed number .

Which is greater  $1\frac{1}{9}$  or  $\frac{5}{3}$ ?

Approach 1 : Convert  $1\frac{1}{9}$  into improper fraction.

$$1\frac{1}{9} = \frac{9+1}{9} = \frac{10}{9}.$$

Now compare the two improper fractions  $\frac{10}{9}$  and  $\frac{5}{3}$  by equalizing the denominator.

$$\frac{10}{9} = \frac{10}{9}$$

$$\frac{5}{3} = \frac{5}{3} \times \frac{3}{3} = \frac{15}{9}$$

Now which is greater  $\frac{10}{9}$  or  $\frac{15}{9}$  ?

$$\frac{15}{9} > \frac{10}{9} \text{ since } 15 > 10.$$

$$\therefore \frac{5}{3} > 1\frac{1}{9}.$$

We can also compare  $\frac{5}{3}$  and  $\frac{10}{9}$  by cross product method.

$$\frac{5}{3} \begin{array}{c} \nwarrow \nearrow \\ \nearrow \nwarrow \end{array} \frac{10}{9} \quad 5 \times 9 = 45; \quad 3 \times 10 = 30$$

Since  $45 > 30$ ,  $\frac{5}{3} > \frac{10}{9}$  or  $\frac{5}{3} > 1\frac{1}{9}$ .

Approach 2 . Convert  $\frac{5}{3}$  into mixed number and then compare.

Now which is greater  $1\frac{1}{9}$  or  $1\frac{2}{3}$  ?

Since the whole number part is the same here, compare the fractional part and decide which is greater using the methods used earlier. We find that

$$1\frac{2}{3} > 1\frac{1}{9} \quad \text{or} \quad \frac{5}{3} > 1\frac{1}{9}$$

*To compare a mixed number and an improper fraction, convert mixed fraction into improper fraction and compare the two improper fractions*

### **Comparison of two mixed numbers**

We compare two mixed numbers by their whole number parts. The one having the greater whole number part is greater. If their whole number parts are the same, we compare them by their fractional parts.

For eg.

i) Which is greater  $15\frac{1}{2}$  or  $18\frac{1}{10}$ ?

Since  $18 > 15$ ,  $18\frac{1}{10} > 15\frac{1}{2}$

ii) Which is greater  $16\frac{3}{4}$  or  $16\frac{5}{7}$ ?

Here the whole number part is the same (=16) Compare the fractional part.

Which is greater  $\frac{3}{4}$  or  $\frac{5}{7}$ ? Make use of one of the methods used earlier

and find out that  $\frac{3}{4} > \frac{5}{7}$  Therefore,  $16\frac{3}{4} > 16\frac{5}{7}$

**Note** . We can also compare two mixed numbers by converting them into improper fractions While introducing the cross product method to compare any two fractions, the definition for 'is greater than' is given. Now go through the following

The fraction  $\frac{a}{b}$  is equal to the fraction  $\frac{c}{d}$  if and only if the product  $ad = bc$ .

Eg. Compare  $\frac{2}{5}$  and  $\frac{18}{45}$ .

Since the products are the same here,  $\frac{2}{5} = \frac{18}{45}$ .

*The fraction  $\frac{a}{b}$  is less than the fraction  $\frac{c}{d}$  if and only if the product  $ad < bc$ .*

For eg. Compare  $\frac{1}{6}$  and  $\frac{2}{3}$ .

$$\frac{a}{b} = \frac{1}{6} \qquad \frac{c}{d} = \frac{2}{3}$$

$$a \cdot d = 1 \times 3 = 3$$

$$b \cdot c = 6 \times 2 = 12$$

$$\text{Since } ad < bc \quad \text{i.e. } \frac{a}{b} < \frac{c}{d}$$

$$\text{i.e. } 3 < 12, \therefore \frac{1}{6} < \frac{2}{3}.$$

From these definitions and properties of equality, we can define the trichotomy property for fractions. Given  $\frac{a}{b}$  and  $\frac{c}{d}$  exactly one of the following holds good

$$\text{i) } \frac{a}{b} > \frac{c}{d}$$

$$\text{or ii) } \frac{a}{b} = \frac{c}{d}$$

$$\text{or iii) } \frac{a}{b} < \frac{c}{d}$$

**Exercise**

**Compare the following fractions using proper symbols (>, =, <).**

1.  $\frac{25}{30}$ ,  $\frac{21}{30}$

2.  $\frac{16}{21}$ ,  $\frac{16}{8}$

3.  $\frac{1}{29}$ ,  $\frac{1}{44}$

4.  $\frac{7}{9}$ ,  $\frac{9}{8}$

5.  $\frac{13}{6}$ ,  $\frac{5}{9}$

6.  $\frac{14}{15}$ ,  $\frac{6}{7}$

7.  $\frac{6}{17}$ ,  $\frac{16}{19}$

8.  $\frac{23}{4}$ ,  $2\frac{7}{8}$

9.  $11\frac{2}{3}$ ,  $15\frac{1}{4}$

10.  $\frac{15}{3}$ ,  $\frac{18}{5}$

Knowing 'is greater than', 'is equal to' and 'is less than' we can now order the set of fractions in the same manner that we ordered the set of whole numbers.

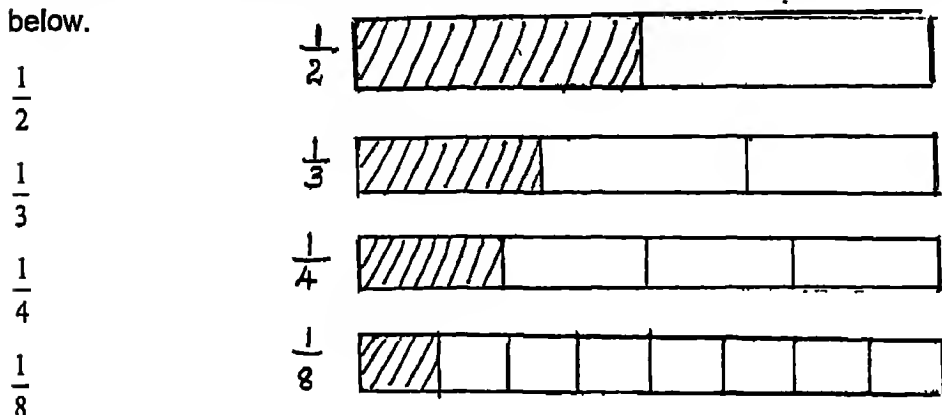
**Activity 1:** Using the fraction disc, do this activity. Pick sectors representing  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , .....  $\frac{1}{10}$  and by observation or by keeping one sector

over the other. Compare two sectors at a time. We find that

$\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \dots > \frac{1}{10}$ . All fractions are only parts of the whole or 1. Using this

fact, we find that  $1 > \frac{1}{2} > \frac{1}{3} > \dots > \frac{1}{10}$ .

**Activity 2 :** Take rectangular paper strips folded into halves, thirds, fourths and eighths. Keep them one below the other in proper alignment as shown below.



Observe that half of the whole is greater than  $\frac{1}{3}$  of the whole,  $\frac{1}{3}$  or the whole is greater than  $\frac{1}{4}$  of the whole and  $\frac{1}{4}$  of the whole is greater than  $\frac{1}{8}$  of the whole

Now put them in descending order using proper sign i.e.  $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{8}$ .

We can also write these fractions in ascending order using proper sign.

i.e.  $\frac{1}{8} < \frac{1}{4} < \frac{1}{3} < \frac{1}{2}$

## DECIMALS

### Unit : Decimal Fractions

#### Introduction

##### Content analysis

1. Concept, understanding decimals through fractions – Expansion of place value chart.
2. Understanding percentage through fractions with denominator 100.
3. approximation (rounding off).
4. Expression of given measurement in higher or lower units through decimals
5. Operations – comparison of decimals – Addition, Subtraction, Multiplication, Division

#### Explanations

1 1 We are familiar with the word 'DECI' generally meaning

Ex .  $\frac{1}{10}$ th of a litre – Decilitre

$\frac{1}{10}$ th of a metre – Decimetre

The idea of decimal was introduced by Simon Stevin, a Belgian in 1585 in the book "La Disme"

**A 'Decimal fraction' is a fraction whose denominator is a power of ten (1, 10, 100, 1000, etc.).**

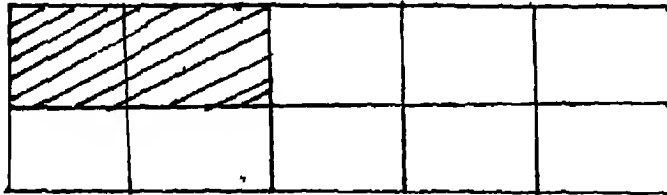
(Note 1 is taken as 1.0, 2 is taken as 2.0.

**The number of digits following the point is called the number of decimal places in the number given.**

Ex The number 3 14 has two decimal places

$\frac{2}{10}$  ,  $\frac{2}{100}$  ,  $\frac{2}{1000}$  are represented in the decimal form as 0 2, 0 02, 0 002 respectively

**Activity .** Each child is asked to bring a card of same size. The teacher asks them to mark ten equal parts on the card as demonstrated by him. The children are asked to shade two parts.

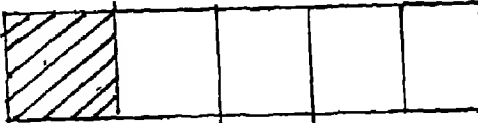


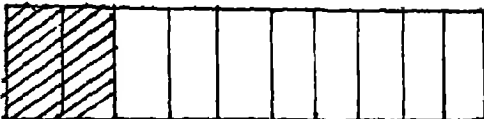
The teacher is suggested to set more number of such activities to children

**Exercise.** The teacher may set exercises containing graded sums on the concept 1.1 for self assessment

### 1.1 Proper and improper fractions as decimals

The teacher should note that a fraction can be converted into a decimal fraction only if the factors of its denominator include 2's and 5's (The other cases are dealt within the concept 3 approximation).

a) Let us take  $\frac{1}{5}$        $\frac{1}{5} =$  

b) Let us divide each part into two  $\frac{1}{5} = \frac{2}{10} =$  

$\therefore \frac{1}{5} = \frac{2}{10} = 0.2$

That is,  $\frac{1}{5} \times \frac{10}{10} = 5 \overline{)1.0} = 0.2$

Let us take an improper fraction  $\frac{5}{2}$ ,  $\frac{5}{2} = 2 \frac{1}{2}$



$$= 2 + \frac{1}{2} = 2.5 \left( \because \frac{1}{2} = 2 \overline{)1.0} = 0.5. \right.$$

1

1

$\frac{1}{2}$

+

+

Similarly,  $\frac{7}{5} = 1\frac{2}{5} = 1 + \frac{2}{5} = 1.4.$

Note that a decimal or a decimal fraction consists of two parts – an integral part (whole number part) and a decimal part (fractional part). The whole number part and the decimal part are separated by a dot called point. eg. 2.5, 1.4.

**Exercise:** Express the following in decimal form :

- a)  $\frac{1}{2}, \frac{1}{5}, \frac{2}{5}, \frac{1}{10}, \frac{3}{10}$  (Ans. 0.5, 0.2, 0.4, 0.1, 0.3)
- b)  $\frac{3}{2}, \frac{8}{5}, \frac{13}{10}$  (Ans. 1.5, 1.6, 1.3).

The teacher will set graded sums on the concept 1.2 as exercises for self assessment.

1.2 Place value and decimals

The knowledge of place value beyond ten in terms of multiples of ten is already familiar

Let us examine if the place values for places less than unit in terms of ten could be considered

100000	10000	1000	100	10	1 unit	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$
						0.1	0.01	0.001	0.0001	0.00001

From units place as you move towards your left, the place value increases by ten times at each level.

As you move from the unit's place towards your right, the place value decreases by ten times at each level.

A 'zero' after 1 in the whole number part increases the value of the number like 1, 10, 100, etc. A 'zero' in the decimal part or a zero immediately after the decimal point decreases the value of the number like 0.1, 0.01, 0.001, etc.

Let us try to make the meaning of a decimal number clear by considering their place values.

$$\begin{aligned}\text{Ex. } 374.52 &= 300 + 70 + 4 + \frac{5}{10} + \frac{2}{100} \\ &= 3(100) + 7(10) + 4(1) + 5\left(\frac{1}{10}\right) + 2\left(\frac{1}{100}\right) \\ &= 3(100) + 7(10) + 4(1) + 5(0.1) + 2(0.01)\end{aligned}$$

**Exercise:** Expand the following numbers indicating their place values.

- (i) 0.3, 0.05, 0.007, 0.17, 0.017
- (ii) 1.15, 21.05, 321.121, 457.025

The teacher will set graded sums as exercises on the concept 1.3 for self assessment.

### 1.3 Expression of a fraction as a decimal.

We know already well that  $\frac{5}{10} = 0.5$ ,  $\frac{5}{100} = 0.05$  and  $\frac{5}{1000} = 0.005$  and

so on. Now let us try to convert  $\frac{1}{4}$  into a decimal.

$$\star \frac{1}{4} = \frac{1 \times 100}{4 \times 100} = \frac{100}{400} = \frac{100/4}{400/4} = \frac{25}{100} = 0.25$$

$$\text{or } \begin{array}{r} 0.25 \\ 4 \overline{)1.00} \\ \underline{8.0} \\ 2.0 \\ \underline{2.0} \end{array} \quad \therefore \frac{1}{4} = 0.25$$

Note. \* 4 is not a factor of 10. If we divide and multiply  $\frac{1}{4}$  by 10 it amounts to  $\frac{10}{40}$  which needs further simplification. However, since 4 is a factor of 100, it is convenient to multiply and divide by 100. The second method is which in brief dividing 1 by 4 that is first method briefed

In a nut shell, the procedure is (i) Divide the numerator by the denominator; (ii) if the whole number part of the numerator is smaller than the denominator or the remainder that is got after 2 or 3 steps of division, proceed by placing a zero after the remainder simultaneously putting the decimal point in the quotient at this stage.

$$\text{Ex. } \frac{13}{4} = \begin{array}{ll} \text{(i)} & 4 \overline{)3.25} \\ \text{(ii)} & \begin{array}{r} 12 \\ 10 \\ \underline{8} \\ 20 \end{array} \\ \text{(iii)} & \underline{20} \end{array} \quad \therefore \frac{13}{4} = 3.25$$

#### Activity 1 .

- i) The children may be asked how a rupee is divided into coins of its fractions 25 paise, 50 paise, 75 paise
- ii) How many 25 paise pieces has a rupee ? Let the children place one rupee coin and 4 pieces of 25 paise coins. The teacher will write on the board that 1 Rupee = 4 pieces of 25 paise coins and proceed that 1 Rupee = 2 pieces of 50 paise coins.
- iii) Make the students know that
  - $\frac{1}{4}$  of a Rupee = 1 piece of 25 paise coin = 0.25
  - $\frac{1}{2}$  of a Rupee = 2 pieces of 25 paise coins =  $0.25 \times 2$

$\frac{3}{4}$  of a rupee = 3 pieces of 25 paise coins =  $0.25 \times 3$

Then  $\frac{1}{4} = 0.25$ ,  $\frac{1}{2} = 0.5$ ,  $\frac{3}{4} = 0.75$

iv) Now it is time for the children to form a conversion table.

Fraction	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
Decimal form of the fraction	0.03125	0.0625	0.125	0.25	0.50 or 0.5	0.75

The children may be asked to prepare such more tables and find the relation between them comparing the fractional part and decimal part of each item.

**Exercise:** Convert the following fractions into decimals.

i)  $\frac{1}{40}, \frac{1}{400}, \frac{1}{1000}$  (Ans: 0.025, 0.0025, 0.00025)

ii)  $\frac{1}{25}, \frac{2}{25}, \frac{3}{25}, \frac{4}{25}$  (Ans: 0.04, 0.08, 0.12, 0.16)

iii)  $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$  (Ans: 0.5, 1.5, 2.5, 3.5)

7 The teacher will set more such sums on the concept 1.4 for better understanding and self assessment.

#### 1.4 Expression of a Decimal as a fraction

Suppose we want to convert 0.325 into a fraction. According to place value.

	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
		0.1	0.01	0.001
0.325 =		0.3	0.02	0.005
0.325 =		$\frac{3}{10}$	$\frac{2}{100}$	$\frac{5}{1000}$

$$\therefore 0.325 = \frac{3}{10} + \frac{2}{100} + \frac{5}{1000} = \frac{300 + 20 + 5}{1000} = \frac{325}{1000}$$

$$\frac{325}{1000} = \frac{13}{40} \text{ (after simplification)}$$

The process involves three steps

- i) Take the number on the numerator.
- ii) In the denominator put 1 for the point and as many zeros as the number of digits on the numerator
- iii) Simplify .

$$\text{Ex. } 0.0004 = \frac{0004}{10000} = \frac{4}{10000} = \frac{1}{2500}.$$

**Exercise:** Convert the following decimal fractions into fractions

$$\text{i) } 0.2, 0.02, 0.002 \text{ (Ans } \frac{1}{5}, \frac{1}{50}, \frac{1}{500} \text{)}$$

$$\text{ii) } 1.2, 1.12, 11.2, 11.02 \text{ ( } \frac{3}{2}, \frac{28}{25}, \frac{56}{5}, \frac{551}{50} \text{)}$$

The teacher will set graded sums on the concept 1.5 as exercises for better understanding and self assessment.

## 2.1 Understanding of percentage through fraction with denominator 100.

$$\text{We know } \frac{3}{4} = 0.75 = \frac{75}{100} = 75\%$$

$$\frac{1}{2} = 0.50 = \frac{50}{100} = 50\%$$

$$\frac{1}{4} = 0.25 = \frac{25}{100} = 25\%$$

$$\frac{1}{20} = 0.05 = \frac{5}{100} = 5\%$$

Let us try to convert percentages into decimal fractions.

$$\frac{1}{2} \% = 0.5\% = \frac{0.5}{100} = 0.005$$

$$1\frac{1}{2} \% = 1.5\% = \frac{1.5}{100} = 0.015$$

$$6\frac{1}{4} \% = 6.25\% = \frac{6.25}{100} = 0.0625\%$$

**Exercise :** Convert the following percentages into decimal fractions

a) 5%, 10%, 15%, 32% (Ans: 0.05, 0.1, 0.15, 0.32)

b)  $\frac{2}{5}\%$ ,  $\frac{3}{5}\%$ ,  $\frac{7}{10}\%$ ,  $\frac{9}{10}\%$  (Ans: 0.004, 0.006, 0.007, 0.009)

The teacher will set sums on the concept 2.1 for better understanding and self-assessment

### 3. Approximation (Rounding off)

**Activity :**

- 1 Children are asked to draw straight line segments of length 2.5 cm, 2.55 cm, 3.6 cm, 3.64 cm. They easily draw 2.5 cm and 3.6 cm long line segments. They fix the length between 2.5 and 2.6 while drawing a line segment 2.55 cm long. That is they draw a line segment which is approximately 2.55 cm long not exactly 2.55 cm long. Similarly they draw approximately 3.64 cm long segment, with the available scale in their instrument box. The length is not precise but approximate.
- 2 The children are asked to bring different circular objects and a thick twine thread.
  - a) Let them find the length of the boundary of the circular objects on a paper.

- b) Let them find the length of the boundary of the circle that is the circumference with the help of the thread and a scale.
- c) Let them find the length of the diameter of the circle by any convenient method suggested by the teacher.
- d) Let them find the ratio between the circumference and the diameter in each case. Let the ratio be  $\frac{C}{D}$ . This ratio is called pi or  $\pi$ . Let them discover that  $\frac{C}{D}$  is a constant.
- e) In each case it is found that the circumference is a bit more than 3 times the diameter in length.

That is  $\frac{C}{D} > 3$  but  $\dots < 3.15$ . This ratio is called pi or  $\pi$  and approximately the value of  $\pi$  is taken as  $\frac{22}{7}$  or 3.1416.

It is interesting to note that our system of notation has not been able to produce a notation which exactly symbolizes the value of the circumference diameter ratio. With the help of a computer, the number has been calculated to 100000 digit places without finding an even division.

3. It is known that a fraction can be converted to a decimal fraction only if the prime factors of its denominator include 2's and 5's (please see 1.2). Fractions like  $\frac{1}{3}$ ,  $\frac{1}{7}$ ,  $\frac{1}{11}$  cannot be converted into terminating decimals but into recurring decimals.

The above circumstances have led a way for resorting to approximations

$$\frac{1}{3} = 0.333\ldots$$

= 0.3 (reduced to one decimal place).

$$\frac{1}{7} = 0.14285714 \ldots$$

= 0.143 (Approximated to three decimal places)

= 0.14 (approximated to two decimal places)

$$\frac{1}{11} = 0.090909.....$$

= 0.091 (reduced to three decimal places).

$$\frac{22}{7} = 3.142857.....$$

= 3.14 (reduced to two decimal places).

### Rules of Approximation:

- i) If the terminating number is less than 5 it is to be rounded off to the previous number or one more than the previous number if the terminating number is more than 5. Ex. 1 0 333 = 0 3, 3.14 = 3.1, 2 3.16 = 3.2, 0 0909 = 0 091.
- ii) If the terminating number is just 5, then the rules stated above cannot be applied, hence a third rule that it is to be rounded to the nearest even number is applied Ex. i) 3 15 = 3 2, ii) 3.45 = 3 4, iii) 0 14285 = 0.1428.

**Exercises:** Round off the following decimals to the two decimal places.

- i) 1.3235      ii) 1 0086      iii) 57.5758
- iv) 63 6445

Ans: i) 1.32, ii) 1 01, iii) 57 58 iv) 63 64

The teacher will set graded sums on the concept 3 for better understanding and self assessment

### 4. Expression of a given measurement in lower or higher units through decimals.

The teacher must see that the children are conversant with tables of different measurements like length, area, volume (capacity), mass, etc. A revision of such tables for recapitulation is necessary.

For an illustration let us consider the measurement of length.

- a) 2 metres and 57 cms = 2.57 metres. Instead of saying 2 metres and 57 cms of cloth is purchased, it is convenient to say 2 57 metres of cloth if purchased.



- b) The distance between two villages Rampur and Bilaspur is 25 km and 37 m. The distance in km is expressed thus

$$25 \text{ km } 37 \text{ m} = 25 \frac{37}{1000} = 25.037 \text{ km}.$$

**Suggested Activity :**

1. The children may be asked to observe carefully the scale in their instrument box. Let them find out how each cm is divided into ten equal parts.
2. The teacher may ask the children to draw a line segment and mark certain length. Later ask the children to express the length in cms or mms
3. The teacher may suggest an outdoor activity. Let the children count how a km is divided by observing the distance between any two consecutive stones marking kms.

The concept may be extended by the teacher for measuring area, volume and mass.

**Exercise:**

- i) Convert the following into metres.  
527 cms, 307 cms, 45 cm, 33 cm and 8 mm  
Ans. (5.27 m, 3.07 m, 0.45 m, 0.33 m)
- ii) Convert the following into km.  
4 Km 320 m, 15 Km 37 m, 598 M  
Ans. (4.32 km, 15.37 km, 0.598 km)
- iii) Convert the following into litres ?  
575cc, 25 decilitre, 3 litres 5 decilitres  
Ans. (0.575 lit, 2.5 lit, 3.5 lit)
- iv) Convert the following into kilograms.  
3 Kg 475 gms, 987 gms, 5 Kg 57 gms.  
Ans. (3.475 Kg, 0.987 Kg, 5.057 Kg)

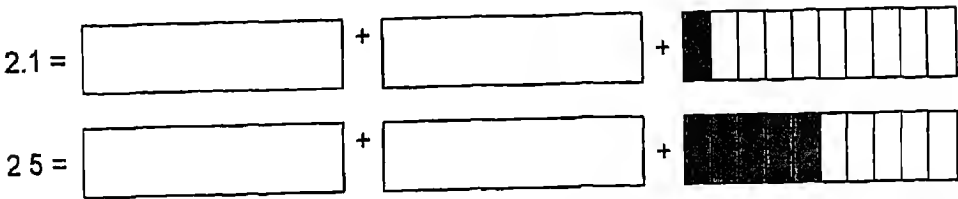
The teacher will set such more graded sums on the concept 4 for better understanding and self assessment.

5. Comparison of decimals, Addition, Subtraction, Multiplication and Division of Decimals.

5.1 Comparison of Decimals

Before getting into operation, the teacher has to get acquainted with the comparison of size of decimals.

a) Out of 2.1 and 2.5, suppose we are to find out which is greater ? Let us examine



On just looking at the illustration we come to know that 2.5 is greater than 2.1

b) Let us take up another case Out of 2.5 and 2.15 which is greater ?

2.5 and 2.15 are written as 2.50 and 2.15. The whole number part is common to both. The remaining part in the first case has 50 parts out of 100 and 15 parts out of 100 in the second. The children will conclude that 2.5 is greater than 2.15.

Activity : The children are asked to bring 40 cards of same size and three coloured pencils – red, blue and yellow. They are asked to make cards into 3 groups the first having  $\frac{1}{4}^{th}$  of the cards, the second having  $\frac{1}{8}^{th}$  of the cards and the remaining as the third.

The teacher asks that what is the number of cards in the first group. The answer is obviously 10. The teacher asks them to colour them red. Next the teacher asks them to colour the second group of cards blue. The next teacher asks them to colour the third group of cards yellow. The teacher asks questions: What is the number of cards in the second group i.e. blue cards ?

The answer is 5

What is the number of yellow cards? The answer is 25.

Which group contains most of the cards ?

The answer is the third or the yellow group. What is the number of cards in the largest group? The answer is 25.

Let us try to find out mathematically. What is  $\frac{1}{4}$ <sup>th</sup> of 40 in decimal fraction?

$$\frac{1}{4} \text{ of } 40 = 0.25 \text{ of } 40 = 10$$

Similarly,  $\frac{1}{8}$  of 40 = 0.125 of 40 = 5.

The remaining i.e

$$40 - 10 - 5 = 25$$

$$1 - \frac{1}{4} - \frac{1}{8} = 1 - 0.25 - 0.125 = 0.625$$

$$\text{Also, } 1 - \frac{1}{4} - \frac{1}{8} = \frac{8 - 2 - 1}{8} = \frac{5}{8} = 0.625.$$

Among 0.25, 0.125 and 0.625 which is greater.

The answer is 0.625.

Suppose there were 1000 cards, 250 of them are red, 125 are blue and 625 are yellow. Yellow cards are more than the remaining.

Among the given decimal fraction, the way of finding the greater decimal is

- i) Bring the decimal fractions to same order without changing their value  
Ex. 0.01, 0.1, 0.001 = 0.010, 0.100, 0.001
- ii) Find out which is greater ?

#### Exercises:

1 Arrange the following in the ascending order

- i) 0.5, 0.25, 0.025, 0.05 (Ans. 0.025, 0.05, 0.25, 0.5)
- ii) 1.3, 1.13, 1.31, 1.03 (Ans. 1.03, 1.13, 1.3, 1.31)

2 Arrange the following in the descending order

- i) 0.02, 0.2, 0.002, 0.0002 (Ans. 0.2, 0.02, 0.002, 0.0002)
- ii) 2.05, 2.5, 0.25, 0.025 (Ans. 2.5, 2.05, 0.25, 0.025)

3. Pick out the greatest and the least decimal fractions among the given.

3.125, 31.25, 312.5, 0.3125 (ans. Greatest 312.5, Least 0.3125)

The teacher will set more number of such numerical sums and word sums on the concept 5.1 for better understanding, application and self assessment

## 5.2 Addition and Subtraction

For any operation, the knowledge of place value is important. The teacher should bear in mind that the interval between any two consecutive places is divided into ten equal parts.

Let us study some examples of addition and subtraction of decimal fractions.

i)  $3.1257 + 15.041$

$$= \begin{array}{r} 3.1257 \\ + 15.0410 \\ \hline 18.1667 \end{array}$$

$$+ 15.0410$$

$$\underline{18.1667}$$

ii)  $1.0089 - 0.9897$

$$\begin{array}{r} 1.0089 \\ - 0.9897 \\ \hline 0.0192 \end{array}$$

$$- 0.9897$$

$$\underline{0.0192}$$

This amounts to that Decimal additions and subtractions are carried out in the same way as in the addition of integers of course with special attention to the decimal point

### Exercise :

i) Find:  $2.5 + 25 + 0.25 + 0.025$  (Ans: 27.775)

ii) Simplify  $183.183 + 18.3183 - 1.83183 + 1831.83 - 0.183183$   
(Ans: 2031.316287)

iii) Sita had 200 rupees with her. She spent 0.25 parts for purchasing book, 0.125 parts for visiting cinema, 0.325 parts for purchasing cosmetics. Find the amount that remained with her (Ans Rs 60/-)

The teacher will set such more graded problems as the concept 5.2, for better understanding, application and self assessment).

### 5.3 Multiplication

Let us study an example

$$0.325 \times 0.024$$

$$\begin{array}{r}
 = \quad 0.325 \quad . \quad . \quad . \quad 3 \text{ decimal places} \\
 \times \quad 0.024 \quad \dots\dots \quad 3 \text{ decimal places} \\
 \hline
 \quad 1300 \\
 \quad 0650 \\
 \underline{0000} \\
 0.007800 \quad . \quad . \quad 6 \text{ decimal places} \\
 \\
 = 0.0078
 \end{array}$$

Explanation : (Reason)

$$\begin{aligned}
 \frac{325}{1000} \times \frac{24}{1000} &= \frac{325 \times 24}{10 \times 10 \times 10 \times 10 \times 10 \times 10} \\
 &= \frac{7800}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = 0.007800 = 0.0078
 \end{aligned}$$

This amounts to that the multiplication of decimal fractions are carried as in the multiplication of whole numbers or integers with due care on the decimal point and the places of decimals in the operation

Note

- i) When two decimal fractions without any whole number part (integral part) are multiplied, the value (magnitude) of the product is less than the value (magnitude) of either multiplicand or the multiplier

$$\text{Ex. } 0.21 \times 0.2 = 0.042$$

- ii) In the case of multiplication of two decimal fractions, one of them having a whole number part, the magnitude of the product is less than the magnitude of either the multiplier or the multiplicand whichever has the whole number part.

$$\text{Ex. i) } 2.1 \times 0.2 = 0.42$$

$$\text{ii) } 0.03 \times 4.3 = 0.129$$

- iii) In the case of multiplication of two decimal fractions having whole number part in both of them, the magnitude of the product depends on the whole number part and is always greater than the magnitude of the multiplicand or the multiplier

$$\text{Ex. } 2.3 \times 3.01 = 6.923$$

**Exercise : Find the product of**

- i)  $2\,5314 \times 0.015$  (Ans. 0.0379710)
- ii)  $471.21 \times 2.5$  (Ans. 1178.025)
- iii)  $0.081 \times 0.37$  (Ans. 0.02997)

The teacher will set numerical word problems on the concept 5.3 as exercise for better understanding and self assessment.

Hint: In the problem, divide Rs.3200 equally among four members. The usual way is  $3200 \times \frac{1}{4} = \text{Rs } 800$ . This could be done using decimal fractions  $3200 \times 0.25 = \text{Rs.}800$ .

5.4 Division. The process of division of decimal fraction varies from the process of integers. Suppose we are to divide 2.4 by 0.15. The dividend has one digit in the decimal place and the divisor has two digits in the decimal place. Let us see

$$\frac{2.4}{0.15} = \frac{2.4 \times 100}{0.15 \times 100}$$

Note that when the numerator and the denominator are multiplied by the same number, the value of the system does not change. The process of division becomes easy and meaningful, if the number of digits in the decimal places of the denominator (divisor) are taken into consideration while converting into whole numbers

$$\frac{2.4 \times 100}{0.15 \times 100} = \frac{240.0}{15.00} = \frac{240}{15} = 16$$

$$\begin{aligned} \text{Ex. 2: } 2.43 \div 0.012 &= \frac{2.43}{0.012} \times \frac{1000}{1000} = \frac{2430.00}{012} \\ &= \frac{2430}{12} = 202\,5 \end{aligned}$$

$$\begin{array}{r} \text{or} \quad 12 \overline{)2430} \quad = 202.5 \\ \underline{23} \phantom{0} \\ 30 \phantom{0} \\ \underline{24} \phantom{0} \\ 60 \phantom{0} \\ \underline{60} \phantom{0} \end{array}$$

Now let us see how the process is employed in simplifications.

$$\begin{aligned} \text{Ex:} \quad & \frac{0.015 \times 2.4 \times 1.21}{1.32 \times 1.5} \\ &= \frac{0.015}{1.5} \times \frac{2.4}{1.32} \times 1.21 \\ &= \frac{0.015 \times 10}{1.5 \times 10} \times \frac{2.4 \times 100}{1.32 \times 100} \times 1.21 \\ &= \frac{0.15}{15} \times \frac{240}{132} \times 1.21 \\ &= 0.01 \times 20 \times 0.11 \\ &= 0.20 \times 0.11 \\ &= 0.220 = 0.22 \end{aligned}$$

This could be done in short as

$$\begin{aligned} & \frac{0.015 \times 2.4 \times 1.21}{1.32 \times 1.5} \\ &= 0.01 \times 0.2 \times 11 \\ &= 0.002 \times 11 \\ &= 0.022^* \end{aligned}$$

Exercise .

1 Divide

- i) 6.25 by 0.125 (Ans: 50)
- ii) 1.01101 by 0.101 (Ans: 10.01)

2 Simplify

$$\frac{1.25 \times 0.6250 \times 6.25 \times 81.81}{2.5 \times 2.25 \times 22.5}$$

(Ans: 3.15625)

\* **Alternative Methods.**

$$1. \quad \frac{0.015 \times 2.4 \times 1.21}{1.32 \times 1.5} = \frac{0.015 \times 24 \times 121}{132 \times 15} = 0.022$$

$$2. \quad \frac{\frac{15}{1000} \times \frac{24}{100} \times \frac{121}{100}}{\frac{132}{100} \times \frac{15}{10}} = \frac{15}{1000} \times \frac{24}{10} \times \frac{121}{100} \times \frac{100}{132} \times \frac{10}{15}$$

$$= \frac{22}{1000} = 0.022$$

**Advantages of Decimal Fractions**

- (i) Helps us to judge the relative sizes of numbers at a glance
- (ii) Simplifies computation to a considerable extent.
- (iii) Handling decimals is much like handling whole numbers, the complications of fractions by-passed.



## SIMPLIFICATION OF EXPRESSIONS INVOLVING MORE THAN ONE ARITHMETIC OPERATION

### Introduction

In earlier units the four fundamental operations namely addition, subtraction, multiplication and division involving whole numbers, fractional numbers or decimals were learnt. There we simplified the expressions by using single operation at a time. Many a time, we came across some numerical expressions with two or more of these operations occurring together. To simplify such an expression we need some rules to get unique solution to a particular problem. Otherwise, we get different solutions by performing the operation in different ways for the same problem as shown below:

Simplify  $8 + 3 \times 4 \div 2 - 1$ .

There are many ways one can simplify the problem. But they all may give different answers.

Way 1 :

$$\begin{aligned}
 &8 + 3 \times 4 \div 2 - 1 \\
 &= 11 \times 4 \div 2 - 1 && \text{(Addition of 8 and 3)} \\
 &= 44 \div 2 - 1 && \text{(Multiplication of 11 and 4)} \\
 &= 22 - 1 && \text{(Division of 44 by 2)} \\
 &= 21 && \text{(Subtraction)}
 \end{aligned}$$

**Order of operation is :** Addition  $\rightarrow$  multiplication  $\rightarrow$  division  $\rightarrow$  subtraction

Way 2 :

$$\begin{aligned}
 &8 + 3 \times 4 \div 2 - 1 \\
 &= 8 + 3 \times 4 - 1 && \text{(Subtraction of 1 from 2)} \\
 &= 8 + 3 \times 4 && \text{(Division of 4 by 1)} \\
 &= 8 + 12 && \text{(Multiplication of 3 and 4)} \\
 &= 20 && \text{(Addition)}
 \end{aligned}$$

**Order of operation is :** Subtraction  $\rightarrow$  Division  $\rightarrow$  Multiplication  $\rightarrow$  Addition

Way 3 :

$$\begin{aligned}
 &8 + 3 \times 4 \div 2 - 1 \\
 &= 8 + 3 \times 2 - 1 && \text{(Division of 4 by 2)} \\
 &= 8 + 3 \times 1 && \text{(Subtraction of 1 from 2)} \\
 &= 8 + 3 && \text{(Multiplication of 3 and 1)} \\
 &= 11 && \text{(addition)}
 \end{aligned}$$

Order of the operation is: Division  $\rightarrow$  Subtraction  $\rightarrow$  Multiplication  $\rightarrow$  Addition

$$\begin{aligned}
 \text{Way 4 : } & 8 + 3 \times 4 + 2 - 1 \\
 & = 8 + 3 \times 2 - 1 && \text{(Division of 4 by 2)} \\
 & = 8 + 6 - 1 && \text{(Multiplication of 3 and 2)} \\
 & = 14 - 1 && \text{(Addition of 8 and 6)} \\
 & = 13 && \text{(Subtraction)}
 \end{aligned}$$

Order of the operation is : Division  $\rightarrow$  Multiplication  $\rightarrow$  Addition  $\rightarrow$  Subtraction [DMAS]. So in the above example, we find different answers to the same problem by following different sequential orders. Therefore, we need to follow some order of mathematical operations which leads to solution. Hence we follow the following order

Division  $\rightarrow$  Multiplication  $\rightarrow$  Addition  $\rightarrow$  Subtraction (DMAS)

Therefore way 4 is the correct method. However, brackets are used to indicate the topmost priority of the operations.

Activity I :

Example :

Ramu had Rs.500/- He purchased 5 mts of cloth at a rate of Rs.75 per mt. How much money he is left with? Teacher asks questions and gets answers to this problem.

1. How much money Ramu had ?
2. How many metres of cloth he purchased ?
3. What is the rate of the cloth per metre ?
4. What is the cost for 5 metres ?
5. How much money he is left with?

According to the above questions, teacher writes expression in order.

$$\begin{aligned}
 & 500 - 5 \times 75 && \text{(Multiplication)} \\
 & = 500 - 375 && \text{(Subtraction)} \\
 & = 125
 \end{aligned}$$

Order of operation is  $M \rightarrow S$  but not  $S \rightarrow M$  (Why?)

### Activity II : Example

A merchant purchased two boxes of apples each containing 40 apples. Out of these, 6 apples were spoiled and he sold the remaining for Rs.5 per apple. How much money does he get ?

Here teacher asks questions according to the operational order and gets answers.

- i) How many boxes of apples he purchased ?
- ii) How many apples are there in each box ?
- iii) How many apples are there in two boxes ?
- iv) How many apples are spoiled ?
- v) What is the number of remaining apples ?
- vi) What is the selling rate per apple ?
- vii) How much money does he get from the apples ?

Teacher writes the expressions according to his questions

$$\begin{aligned}
 &(40 \times 2 - 6) 5 && \text{(Operate bracket i.e. multiply within the bracket).} \\
 &= (80 - 6) 5 && \text{Subtract.} \\
 &= 74 \times 5 && \text{Multiply} \\
 &= 370
 \end{aligned}$$

Order : (Bracket : within that  $M \rightarrow S$ )  $\rightarrow$  Multiplication

i.e.  $(B : M \rightarrow S) \rightarrow M$

So in the above activities, we find Bracket is to be operated first i.e. the operations within the brackets are operated – subtraction after multiplication. Therefore, the order to be operated is Bracket  $\rightarrow$  Multiplication  $\rightarrow$  Subtraction

In simplifying an expression we use the order of operation 'BODMAS'.

For example

$$\begin{aligned}
 &(8 + 3) \times 4 + 2 - 1 && \text{(Operate bracket)} \\
 &= 11 \times 4 + 2 - 1 && \text{(Divide)} \\
 &= 11 \times 2 - 1 && \text{(Multiply)} \\
 &= 22 - 1 && \text{(Subtraction)} \\
 &= 21
 \end{aligned}$$

Order is : Bracket  $\rightarrow$  Divide  $\rightarrow$  Multiply  $\rightarrow$  Subtract.

In the above example without brackets, we get the answer 13.

In any mathematical expression, the part inside the bracket is worked out first irrespective of fundamental operations. If more than one operation is there within the brackets; then we follow the usual order DMAS, in respect of operations within the brackets also.

In an expression, there may be more than one pair of brackets also. In such a case, the innermost bracket is evaluated first.

### Notations for different types of brackets :

1. [ ] - Big bracket or Box bracket.
2. { } - Flower bracket or braces
3. ( ) - Common bracket or parenthesis
4. — - Bar bracket or Vinculum

For example,

$$\begin{aligned}
 & [ 63 - \{ 7 + 2 \times (8 - 5) \} ] \div \frac{2}{5} \times \frac{1}{3} \\
 &= [ 63 - \{ 7 + 2 \times 3 \} ] \div \frac{2}{5} \times \frac{1}{3} && \text{(Operation of parenthesis)} \\
 &= [ 63 - \{ 7 + 6 \} ] - \frac{2}{5} \times \frac{1}{3} && \text{(Multiplication within braces)} \\
 &= [ 63 - 13 ] - \frac{2}{5} \times \frac{1}{3} && \text{(Addition, operation of braces)} \\
 &= 50 - \frac{2}{5} \times \frac{1}{3} && \text{(Subtraction, operation of braces)} \\
 &= 50 \times \frac{5}{2} \times \frac{1}{3} && \text{(Division of 50 by 2/5).} \\
 &= \frac{250}{6} && \text{(Multiplication)} \\
 &= 41 \frac{4}{6} \\
 &= 41 \frac{2}{3}
 \end{aligned}$$

Sometimes in an expression, there may come operations like 'of'. For example,  $\frac{1}{2}$  of  $\frac{1}{6}$  means  $\frac{1}{2} \times \frac{1}{6}$ .

Example :  $\frac{1}{2}$  of  $\frac{1}{6} (2 + 7 - \frac{1}{2} \times \frac{3}{5} \div \frac{5}{8})$

In this case we follow the order Bracket  $\rightarrow$  Of  $\rightarrow$  Division  $\rightarrow$  Multiplication  $\rightarrow$

Addition  $\rightarrow$  Subtraction (BODMAS)

- 1 B – Deal with inside Bracket
2. O – Work out 'Of' (if any)
- 3 D – work out division
- 4 M – Work out Multiplication
- 5 A – work out Addition
6. S – Work out Subtraction

Therefore, when using different signs together in a numerical expression the word "BODMAS" is helpful.

Example.

$$\frac{1}{3} \text{ of } (12 - 7) + 56 \div (9 - 5)$$

$$= \frac{1}{3} \times (5) + 56 \div (4) \quad (\text{Rule 1})$$

$$= \frac{1}{3} \times 5 + 56 \div 4 \quad (\text{Rule 2})$$

$$= \frac{5}{3} + 14 \quad (\text{Rule 3})$$

$$= \frac{5+42}{3} \quad (\text{Rule 5})$$

$$= \frac{47}{3}$$

$$= 15\frac{2}{3}$$

Exercises :

Simplify .  $73 - \{ 65 \times 2 - (14 \times 5 - 8) \}$

$$\text{i) } 6 \times \{ 4 + 2 - \frac{1}{5} \text{ of } 8 \div 2 \}$$

$$\text{ii) } (15 + 5) - 2 + 6 \times 400 \div \frac{1}{2} \text{ of } 20(13 - 4 \times 2)$$

$$\text{iii) } 2\frac{1}{2} \times 1\frac{1}{3} - \frac{3}{5} \div 1\frac{4}{11}$$

$$\text{iv) } 1\frac{1}{2} \div (2\frac{4}{9} \times 1\frac{4}{11})$$

$$\text{v) } \frac{3\frac{1}{3} + 1\frac{1}{4} \times \frac{2}{5}}{2\frac{2}{5} - 1\frac{1}{4}}$$

$$\text{vi) } \frac{7}{8} \times (3\frac{1}{4} - 2\frac{2}{7}) + \frac{1}{2}.$$

$$\text{vii) } 0.6 \text{ of } (1.8 + 1.2) - 0.8 \times (2.25 - 1.20) + 1.5$$

$$\text{viii) } 1.6 \{ 0.4 + 12.2 - 0.5 \text{ of } 0.8 \div 0.5 \}$$

Note While simplifying an expression we use the order 'BODMAS'

Exactly the same approach is followed for simplifying an expression involving only division

For example :

$$\text{i) } 8 - 3 - 2 = 5 - 2 \text{ (3 is subtracted by 8)}$$

$$= 3$$

$$\text{ii) } 250 \div 10 - 5$$

$$= \frac{250}{10} \div 5$$

$$= 25 \div 5$$

$$= 5$$

## ZERO AND ITS PROPERTIES

### Introduction

The zero is surely one of the greatest inventions of human history. The Indians are usually credited the first to conceive the concept of zero around the 7<sup>th</sup> century although the Babylonians must have had some knowledge of it. In the sequence of inventions, the counting numbers or natural numbers were first to be invented and much later zero was invented. May be for this reason that zero is not considered as a natural number. The Egyptians and Romans apparently did not recognize the need for a symbol for zero as they felt that the word "nothing" was sufficient. The Arabs are credited with introducing the symbol '0' for zero. However, there are also reports that the invention of zero took place by gradual growth of culture and not by sudden discovery.

1. **Concept of Zero:** The absence of all of the objects that have been under consideration is recognized as zero.

Before starting the activities to introduce the concept of zero, the teacher should ensure that children have already understood the addition and subtraction of natural numbers.

Teacher can begin with an activity of the following type. Teacher may take say 6 objects in a plate and then interact with the class.

T : How many objects are there in the plate ?

S : Six objects

T : Take one from them.

T : How many are left ?

S : Five are left.

This way he continues until one object is left.

T : How many are left now ?

S : One is left.

T : Take this object also.

T: How many are left now ?

S: No fruits are left now.

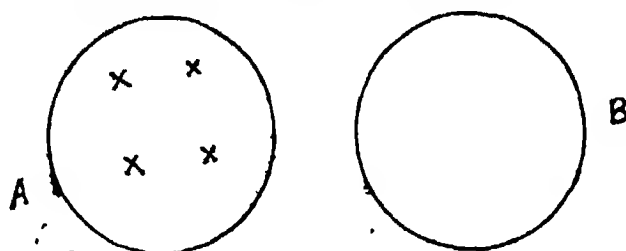
After conducting few such activities with varying number of objects, teacher can explain them that when no objects are left then we say that there are zero number of objects. Since taking away of objects is synonymous with subtraction, teacher can use such activities to infer that for any natural number  $n$ ,  $n - n = 0$ . However, it could be advantageous for the teacher to use the same activity to make children understand that any natural number is greater than 0.

## 2. Properties of zero

i) Any number  $+ 0 = 0 + \text{any number} = \text{that number}$

Ex :  $0 + 4 = 4$

Then 4 and 0 can be represented by groups of objects as shown below.



T: How many objects are there in A ?

S: 4 objects are there in A.

T: How many are in B ?

S: No objects are there in B

T: How do you write this by using numbers ?

S: There are 0 objects in B.

T: Altogether how many objects are there in groups A & B ?

S: There are 4 only as there are no objects in B.

T: If objects present in A are added to objects present in B, then how many objects do you get ?

S: Four.

T: Answer is correct but how do you express it mathematically.

S:  $4 + 0 = 4$ .



Teacher may give some more such activities/examples and then lead the children to conclude that any number added to zero or zero added to any number gives the number itself. The above type of activity may be *modified* by the teacher to lead the students to conclude that any number  $- 0 =$  that number

**The teacher should ensure that children have already understood the addition of natural numbers and subtraction of natural numbers.**

ii) Any number  $\times 0 = 0 \times$  any number  $= 0$

Ex Consider the multiplication table of 5 in the reverse order as follows :

$$5 \times 5 = 25; \quad 5 \times 3 = 15$$

$$5 \times 4 = 20, \quad 5 \times 2 = 10$$

$$5 \times 1 = 5$$

Here the child may be made to observe the pattern that every time to write the table in the reverse order, one has to subtract 5 from the previous answer. After reaching the step  $5 \times 1 = 5$  to find  $5 \times 0$  by observation,  $5 \times 0 = 5 - 5 = 0$

Teacher after considering several such examples should conclude that any number  $\times 0 = 0$ . For the case  $0 \times n = 0$ , it may be easy to construct the multiplication table of 0 by treating multiplication as repeated addition and thereby lead the children to conclude that  $0 \times n = 0$

Finally, we conclude this unit on zero by explaining that division by zero is not possible. Since division in the initial stages is taken as repeated subtraction, it may be easier to explain this by using this notion of division. Recall that when we say  $4 - 2 = 2$  we may mean that 2 subtracted twice from 4 yields zero. Similarly  $6 \div 2 = 3$  can mean 2 subtracted thrice from 6 yields zero. If we consider say  $4 \div 0$  then one must find how many times 0 is to be subtracted from 4 to get 0 but any number of times one may subtract 0 from 4 but still one gets only 4. Hence, we conclude that it is not possible to divide a non-zero number by 0. But then this line of thinking makes one conclude that  $0 \div 0 = 0$  as one does not have to subtract 0 any number of times from 0

to get 0. Yet another possibility is that  $0 \div 0$  may be taken as 1 as  $x \div x = 1$  for any  $x \neq 0$ . At this stage teacher should however straight away tell the children that  $0 \div 0$  is neither 1 nor zero and about it they will study at the higher classes in the college that it has no meaning.

## WHY THE PRODUCT OF TWO NEGATIVE NUMBERS IS POSITIVE ? (OR WHY (-2) INTO (-3) IS PLUS 6?)

The question, whenever asked, has puzzled the students and teachers of elementary mathematics. If  $m$  is a natural number and  $n$  is a whole number (or a real number), the product of  $m$  and  $n$  (i.e.  $m \cdot n$ ) can be thought as "m times n" (i.e. adding  $n$  to itself  $m$  times). So  $2 \cdot 3$  can mean adding 3 to itself two times.  $2 \cdot (-3)$  can be interpreted as  $(-3)$  added to itself two times. But this argument will not hold good for the product of two negative numbers. While considering  $(-2) \cdot (-3)$ , it makes no sense to think of adding  $(-3)$  to itself  $(-2)$  times. Even some arguments like "moving backwards" or "negative voting" are not convincing.

The rule that the product of two negative numbers is positive (i.e.  $(-m) \cdot (-n) = m \cdot n$ ) is often called the 'the Law of signs'. If a person has already learnt that

- i)  $(a + b) \cdot c = a \cdot c + b \cdot c$  for any three numbers  $a, b, c$  (Distributive law)
- ii)  $(y + (-y)) = 0$  for any number  $y$  (Additive Inverse)
- iii)  $z \cdot 0 = 0 \cdot z = 0$  for any number  $z$ .

the following proof of Law of Signs is a convincing answer to the question "Why the product of two negative number is positive ?". Perhaps no other arguments are as convincing as this is

### Proof of Law of Signs

Let  $m$  and  $n$  be any two numbers (real)

Let  $X = mn + (-m) \cdot n + (-m) \cdot (-n)$

Considering the first two terms of the R.H.S, we have

$$\begin{aligned}
 1 \quad X &= [m + (-m)]n + (-m) \cdot (-n) && \text{(i) above} \\
 &= 0 \cdot n + (-m) \cdot (-n) && \text{(ii) above} \\
 &= 0 + (-m) \cdot (-n) && \text{(iii) above} \\
 &= (-m) \cdot (-n) && \text{(definition of zero)}
 \end{aligned}$$

Considering the last two terms of the R.H.S, we have

$$\begin{aligned} 2. \quad X &= m. n + -m [n + (-n)] \\ &= m. n + -m \{0\} \\ &= m. n + 0 && \text{(i, ii, iii above)} \\ &= m. n \end{aligned}$$

**Thus from 1 and 2,**

$$(-m)(-n) = m \cdot n$$

Hence the proof. QED (Quite Easily Done)

Since the primary students of Fourth level have a fairly good knowledge of (i), (ii) and (iii) above, the proof should not have problem of understanding for a teacher or a student of elementary mathematics. This should also put an end to authoritative communication from teachers to students

Here m, n are any two numbers. If we replace m, n by 2, 3 we got the proof for the statement "(-2) into (-3) is plus 6"

### Addition table

Every digit of a natural number is a member of  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ . When we add two natural numbers, we first add the digits in one's position of the numbers, then the digits in ten's position and so on. For example, let us add 123 and 87. Here 3 and 7 are respectively in one's position of 123 and 87 and their sum is 10 which has zero 'ones' and one 'ten'. The zero will be placed in the one's position of the sum of 123 and 87. The one ten is carried to the sum of 2 and 8 which are in the ten's positions and the total of all the tens is 11 which has one 'ten' and one 'hundred'. The one ten is placed at the ten's position of the sum of 123 and 87. The process continues. The following explains this better:

	Hund	Tens	Ones
	1	1	
	1	2	3
+		8	7
	2	1	0

Thus the process of addition of any two natural numbers boils down to adding two numbers in  $\{0,1,2,3,4,5,6,7,8,9\}$ . Since zero plus any number is the number itself, we construct the basic addition table involving only the nine numbers  $\{ 1, 2, \dots,9\}$

The Basic Addition Table

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

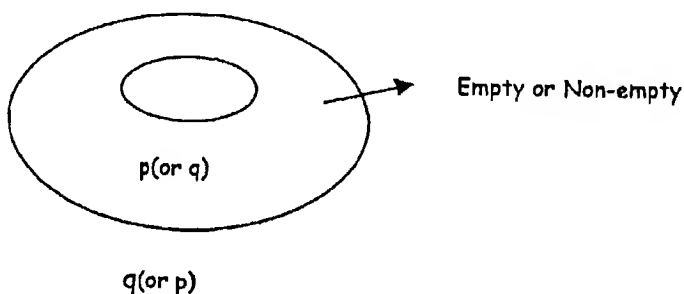
From the table, it is clear that there are in all 45 distinct pairs of numbers who sum needs to be remembered. Thus effectively addition of any two numbers, however big they are, reduces to atmost 45 additions in the *shaded* portion of the Basic Addition Table. A primary school teacher could develop an activity to remember these 45 additions through a song or some other means. However, surprisingly, in primary schools, there is no stress on addition table at all though multiplication table is taught extensively The addition table will considerably reduce the mistakes committed in addition by primary school students

## VENN DIAGRAM AND ITS IMPORTANCE AS A TOOL FOR TEACHING AND LEARNING OF MATHEMATICS

John Venn introduced Venn diagrams in 1880. A Venn diagram represents pictorially interrelations among well defined properties/collections each of which is denoted by a closed region without holes. A well defined property is a property which is either true or false, not both. Though there are other diagrams like line diagram, directed graph, etc to illustrate relationships, Venn diagram has an advantage of *space* over the others.

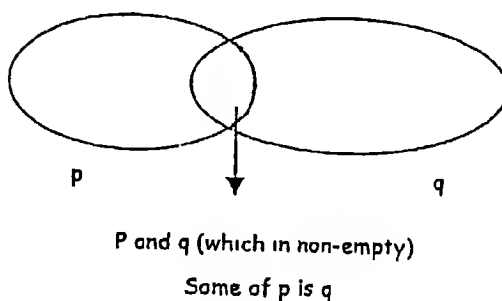
Given two well defined properties  $p$ ,  $q$ , the possible relations between them can be represented by Venn diagram in one of the following ways.

a)

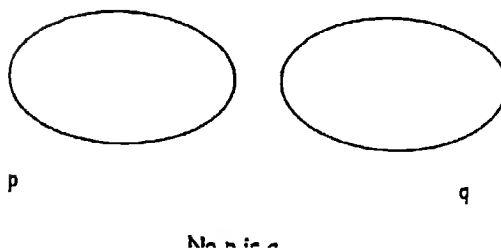


'  $p \Rightarrow q$  ' or '  $p$  is  $q$  ' or '  $p$  is part of  $q$  '

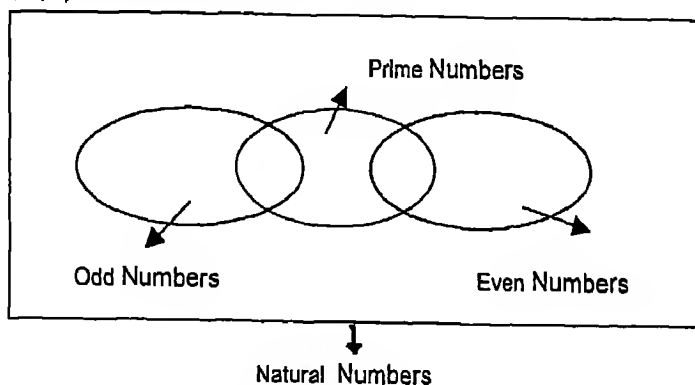
b)



c)



For example, Venn diagram of the properties (i) odd numbers, (ii) Even numbers, (iii) Prime numbers is as follows



Here one can see (in one sight) clearly that no odd number is even, some odd numbers are prime, some even numbers are prime, there are odd numbers which are not prime, there are even numbers which are not prime, every odd number or even number or prime number is a natural number

For example, '2 is a *natural* number' is a property which is well defined but 'Sky has *many* stars' is not a well defined property

Venn diagrams could best be employed in

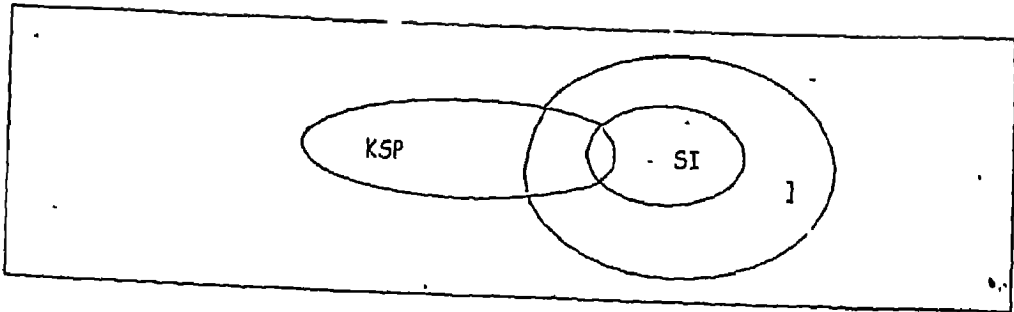
- Better understanding of mathematics because of their visual effect,
- Providing clarity in teaching-learning of mathematics,
- Finding inter-relations among mathematical properties holistically and accurately, and better analysis of the properties

**An example of Venn Diagrammatic Representation Approach (VDRA) experimented with a batch of V grade students, DMS, RIE, Mysore.**

Find interrelations (mathematical) among the following.

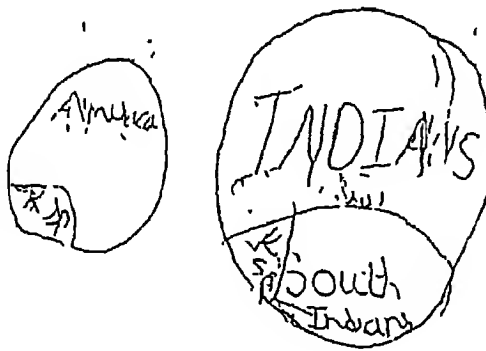
South Indians (SI), Indians (I), Kannada Speaking People (KSP)

Venn Diagrammatic Representation (VDR) of the interrelations is as follows.



One can see that the Venn diagram represents all possible interrelations, whereas in other methods, there is a high chance of missing a relation. The experiment also showed that the V grade students understand Venn diagrams easily.

**An Example of Creativity shown by the child of V grade (Jagdish) during Experiment**



**Note:** Here Venn diagrams are not a part of curriculum of Mathematics at Primary level, but play a role of 'Methodology of teaching Mathematics'.



## **PRIMES**

### **Introduction**

Primes (or prime numbers) are the building blocks of number spectrum. As the name stands they have prime importance in Mathematics

At primary stage it is just given a passing reference in the name of definition and then it is just ignored. As a result, the young children do not get the feel for primes. They are simply deprived of the power and the beauty of primes. Prime factorization is an offshoot of primes leading to the Fundamental Theorem of Arithmetic (also called the unique factorization theorem). It has long been proved that there are infinitely many primes. But there are only a finite number of known primes. Every year mathematicians are discovering newer primes. Any one who discovers a new (of course a large one) prime is rewarded. In some other countries, a new prime discovered is a national property. The main difficulty with primes is that there seems simple pattern at all unlike other mathematical concepts. There is no formula to produce primes.

In this write-up, we desire that the teachers should by all means ensure that in the young minds the concept of prime is set right. A few activities, games and exercises are suggested. The teachers should not only judiciously make use of them but also work out similar (or even better) activities of their own and make learning joyful. It may however be noted that the activities are meant for the students but the teachers should set the activities and let the students explore themselves. The teacher should just play the role of a facilitator but at no instance, he/she should spoon-feed the kids lest all their creativity will vanish into air.

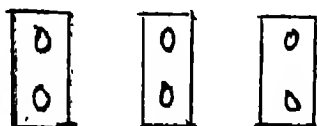
### **Activity 1 : Building the concept of prime**

The teacher may take a number of beads or marbles (or even small stones would do). The children may first be given a specified number (say 6)

of beads and asked to put them in number of groups (containing equal number of beads). The children may come out with the following possibilities.

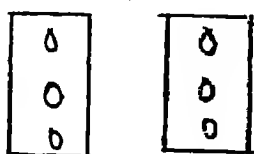
1.

3 groups of 2.



2

2 groups of 3.



3.

6 groups of 1



4

One group of 6



7

Then the teacher put a constraint namely,

- i) In each group there must be more than one bead
- ii) There must be more than one group.

Then the children will discard the 3<sup>rd</sup> and 4<sup>th</sup> way mentioned above. Now the teacher may ask them to do the same for 7 beads. After trying for a while, the kids may express their inability to do so. At this stage, the teacher may tell them that there is no need to feel sorry. In fact, no one can do it as it is impossible. Having said this the teacher may assign the children the task of identifying those numbers (between 2 and 20) for which the 'grouping' is not possible. After testing each number the children will come out with the answer

2,3,5,7,11,13,17,19

Acknowledging their success, the teacher may then say these numbers are special so they deserve a distinct name and declare that they are called 'prime numbers'. Then the teacher may ask the following questions.

1. What is special about prime numbers ?
2. Exactly how many factors (divisors) any prime has ?
3. We call the numbers greater than 1 which are not prime as composite. How many factors can a composite number have ?

The children may be encouraged to try with different numbers. While attempting these questions, the kids may arrive at the following conclusions.

1. A prime has exactly two factors.
2. A composite number has more than two factors

Then the teacher may ask the children whether 1 can be called a prime or a composite. Sooner the kids will realize that 1 satisfies neither of the conditions mentioned above. So they will be convinced that 1 is neither prime nor composite.

### **Activity 2 : Juxtaposing a digit to a given single digit number to obtain a prime.**

The teacher may ask any child to put any digit on the right of a given digit say 4, so that the resulting number becomes a prime. Eg  $4 \rightarrow 43$ , a prime (by juxtaposing 3 on the right of 4).

The children may come up with different answers such as 41, 47, etc. Then the teacher may suggest the children to play this game among themselves. This way they will be get used to prime numbers. They may also, after playing a while, on their own try to juxtapose on the left (e.g.  $3 \rightarrow 23$  etc).

### **Activity 3 : Adding a prime to get a prime**

The teacher may pose to the children the following questions. What prime should be added to 22 so as to get a prime ?

One may come up with the answer such as  $22 + 7 = 29$  or  $22 + 19 = 41$ , etc.

**Activity 4 : Subtraction of a prime from a given number to get a prime.**

This is quite similar to the previous activity (No.3).

**Activity 5 :** Our last activity is in the form of a game which may be called 'Factor game'. This game is highly interesting and addictive. While playing this game, the children would get the feel of the importance of primes. Surprisingly, in the game, there is no mention of prime at all. Before introducing the game, the teacher should ensure that the children are well acquainted with factors (divisors) of a number for this game. The teacher may prepare a few cards (say 20) and mark them 1,2,3,...20. This is all we need to play. It is a single-player game but played against imaginary player (may be called Mr or Ms X).

Initial position of the game is

										Upper box

Move – 1 . (16 is picked up).

Box-P

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
16					1	2	4	8	

Box-X

Move - 2 . (20 is picked up).

		3		5	6	7		9	10
11	12	13	14	15	16	17	18	19	20
16					1	2	4	8	
20					5	10			

Move – 3 . (18 is picked up)

		3			6	7		9	
11	12	13	14	15		17	18	19	
16					1	2	4	8	
20					5	10	9		
18					3	6			

Move – 4 . (14 is picked up)

		7		11		12		13	
14		15			17		19		
16				1	2	4	8		
20				5	10	9			
18				3	6				
14				7					

Final Position

16				1	2	4	8			
20				5	10	9				
18				3	6					
14				7						
				11	12	13	15	17	19	

Total :  $16 + 20 + 18 + 14 = 68$

X-total :  $1 + 2 + 4 + 8 + 6 + 9 + 3 + 7 + 11 + 12 + 13 + 15 + 17 + 19 + 5 + 10$   
 $= 142$

So player X is the winner.

When the game is played for the first time, it would appear that player X will always win. But after playing a number of times, the kids may soon realize that it is possible to win provided players played carefully. During the course of playing many times, they will realize the role of primes in this game. But the teacher should not, at any stage, mention the role of prime in this game. The children should not be deprived of the opportunity to discover it. However the teacher may ask the questions such as :

1. How many times can you choose primes ?
2. In order to win when should you pick a prime ?
3. If you did not pick a prime in the first move, can you pick a prime in the next move ?
4. Which prime should you pick for maximum gains ?

The teacher may first play the game himself/herself and chalk out some strategy to win and based on that he/she may frame some more questions. The teacher may also try to devise some more games where primes indirectly play a role.

Exercises : The exercise should first be tried by the teacher and then posed to the children (not all at a time).

1. What is the smallest prime ?
2. What is the smallest odd prime ?
3. Is there any even prime ?

4. Are all primes odd ?
5. Can two primes differ by one ?
6. Are all odd numbers prime ?
7. List 8 prime pairs such that each number of any pair differ by just two.
8. Can sum of two primes be another prime ?
9. Can a prime be a square ?
10. How many primes end in 5 ?
11. Can a prime be divisible by a square other than 1 ?
12. List all the primes between 10 and 100 whose both the digits are primes.
13. List 8 pairs of primes whose sum is also a prime
14. The primes, 3,5,7 are such that  $5 - 3 = 2$ ,  $7 - 5 = 2$ . Can you find another set of 3 such primes ?
15. Take any even number say 24. We note that  $24 = 11 + 13$  i.e. 24 can be expressed as a sum of two primes. Try expressing 10, 40, 18, 22 and 48 as a sum of two primes
16. Is there an even number which cannot be expressed as a sum of two primes ?
17. Find a prime between 40 and 80. Can you always find a prime between any number above 10 and its double.
18. Is there any number such that between that number and its double, there exists no prime.
19. Observe that
 

$2 \times 3 + 1 = 7$	a prime
$2 \times 5 + 1 = 11$	a prime.
$2 \times 11 + 1 = 23$	a prime

Find 5 more primes such that 2 times that prime plus 1 is a prime.
20. Find 5 primes such that prime plus 1 is a composite.

**Answers :**

1. 2;                      2. 3 ;    3. Yes, 2.                      4. No because 2 is a prime.
5. Yes eg. 2 and 3    6. No e.g. 9 is not a prime.
7. (3,5), (5,7), (11,13), (17,19), (29,31), (41,43), (59,61), (101, 103)
8. Yes,  $2 + 3 = 5$ ,  $2 + 5 = 7$  etc.
9. No.                      10. Only one viz. 5
11. No                      12. 23, 37, 53, 73.
13. (2,3), (2,5), (2,11), (2,17), (2, 29), (2,41), (2,59), (2,101)
14. No (why?)
15.  $10 = 3 + 7$ ,  $18 = 11 + 7$ ;  $22 = 11 + 11$ ,  $48 = 41 + 7$
16. Yes, 2 cannot be expressed as the sum of two primes
17. 41 or 43 etc. Yes.
18. No, between 1 and 2 there is no prime
19.  $2 \times 23 = 47$ ,  $2 \times 29 = 59$ ,  $2 \times 41 = 83$ ,  $2 \times 53 = 107$ ,  $2 \times 59 = 109$ .  
(Many more such examples are there).



## LOWEST COMMON MULTIPLE OF NUMBERS AND FACTORS

### Introduction

Working with fractions often requires us to find the least common multiple of a set of natural numbers. In fact, we use the concept of Least Common Multiples in addition, subtraction and comparison of fractions.

There is no difficulty while adding, subtracting or comparing the fractions, when the denominators of the given fractions are equal. But the difficulty arises, when the denominators of the given fractions are different. So we make the denominators equal. We choose a number which is divisible by all the denominators of the fractions. So, we choose **the least common multiple of** all the denominators. That is, a Lowest Common Multiple.

### Concepts :

Multiples, common multiple, lowest common multiple.

**Notations:** Least common multiple L.C.M.

### Definitions

1. **Multiple** A multiple of a given number is a product of the given number and any natural number as a factor.
2. **Common Multiple** A common multiple of two or more numbers, is a multiple of all the given numbers.
3. **Least common multiple** The smallest of all the common multiples of the given numbers is their least common multiple.

Examples :

Do not get carried away by the "Least" in the term L.C.M. because L.C.M. is never smaller than the greatest of the given numbers, whose L.C.M. has to be found out.

To understand the concepts of L.C.M Now let us see the concepts, multiple, common multiple.

### **Multiples :**

In the third standard the students have learnt about the multiplication tables and are able to write the multiplication tables from 2 to 10. The teacher has to remind the multiplication tables

#### **Multiplication Table (1 to 10)**

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Now consider the table of 2, whose elements are 2,4,6,8,10,12,14,16,18,20

Here 2 contains 2 one time  $2 \times 1 = 2$

4 contains 2 two times  $2 \times 2 = 4$

6 contains 2 three times  $2 \times 3 = 6$

8 contains 2 four times  $2 \times 4 = 8$

...

So, 2,4,6,8, .. are obtained on multiplying 2 with 1,2,3,4, 10 respectively. Therefore, 2,4,6,8, are multiples of 2.

To get the multiples of a number, we have to multiply that number with 1,2,3,4,

Activity

Here is an activity for the students. The students have to complete the table of multiplication.

×	1	2	3	4	5	6	7	8	9	10
1	1									
2	2	4								
3	3	6	9							
4	4	8	12	16						
5	5	10	15	20	25					
6	6	12	18	24	30	36				
7	7	14	21	28	35	42	49			
8	8	16	24	32	40	48	56	64		
9	9	18	27	36	45	54	63	72	81	
10	10	20	30	40	50	60	70	80	90	100

Here students have to fill the blanks in two ways

1 way

Question. When you go on filling the numbers across the row, what do you observe ?

Ans We write the multiples of the corresponding rows

So we can write any number of multiples (by simply adding) or by skipping the interval of row/column

Activity

The teacher distributes flash cards (1 – 30) among the children, and they are told to raise their hands when asked for the multiple of a number and show the respective number i.e flash card So the teacher says – "Show multiples of 2?"

The students having cards 2,4,6,.. 30 raise their hands, so the teacher, with explanation writes the multiples of 2 on blackboard.

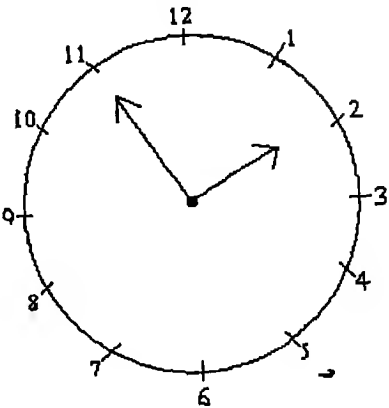
Multiples of 2 . 2,4,6,8 , . 30.

In a similar way, teacher asks "Show multiples of 3?".

The students having flash cards bearing no. 3, 6, 9, 12,...30 raise the hands and the teacher writes on the blackboard.

Thus the children identify numbers and their multiples

Activity



Teacher shows a model of a clock.

- 1. One student has to bring the hour hand at any position, say 2.
- 2. Other student has to bring the minute hand to any position say 11.
- 3. The third student has to say the product. By changing the position of minute hand, keeping hour hand constant we can get the 12 multiples of a number and then by changing the hour hand we can get the multiples.

Thus by this activity, we can develop the multiples of 12, upto 12 digits  
 $12 \times 12 = 144$ .

Activity

APRIL 2003

SUN		6	13	20	27
MON		7	14	21	28
TUE	1	8	15	22	29
WED	2	9	16	23	30
THU	3	10	17	24	
FRI	4	11	18	25	
SAT	5	12	19	26	

Teacher shows the calendar of April 2003 – and asks the following questions.

1. What do you observe in this calendar ?
2. Observe the dates of Sundays?
3. What is special about the dates of Sundays?

Ans: They are multiples of 7.

Activity : Game Guess the number.

The Rules of the Game are :

- \* You cannot “assume” prime nos and the numbers should be between 1 to 100 only.
- \* You can ask only 10 questions
- \* Initially you will have 10 points. For every question asked, you will lose 10 points.
- \* Those who have more points are the winners.

Sample game:

Suppose you have presumed 40 Your probable questions are

	A	Ans	B
1.	Is it multiple of 10 ?	Yes	10
2.	Is it below 50?	Yes	10
3.	Is it multiple of 4?	Yes	10

The number in multiples of 10 and below 50  $\Rightarrow$  10, 20, 30, 40 and multiple of 4  $\Rightarrow$  20 or 40.

4. Is it multiple of 8 ? Yes

$\therefore$  The number is 40 i.e. presumed number.

So A has 60 points and B has 40 points. A has won. So in this way teacher can develop multiple concept.

### Common Multiples

One ice cream van visits Radha's neighbourhood after every 4 days and another ice cream van visits her neighbourhood after every 5 days during the

summer. If both the vans visited today, when is the next time both the vans will visit on the same day ?

Solution:

Ice cream van	Days of visit
First (every 4 <sup>th</sup> day) . Multiple of 4	4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44....
Second van (every 5 <sup>th</sup> day) Multiple of 5	5, 10, 15, 20, 25, 30, 35, 40,.....

∴ Both the ice cream vans visit (a common day) her neighbourhood on 20<sup>th</sup> day and 40<sup>th</sup> day.

∴ 20 is a common multiple of 4 and 5.

1. Example : Consider the two numbers 4 and 8.

The multiples of 4 are : 4, 8, 12, 16, 20, 24, 28, 32, ...

The multiples of 8 are : 8, 16, 24, 32, 40, 48, ...

The common multiples of 4 and 8 are : 8, 16, 24, 32,....

Now 8 contains 8 one times and 4 two times

16 contains 8 two times and 4 four times.

24 contains 8 three times and 4 six times.

2. Example . Consider the numbers 6, 9, 15.

Multiples of 6 : 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90..

Multiples of 9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90,.

Multiples of 15: 15, 30, 45, 60, 75, 90, 105,....

The common multiples of 6, 9, 15 are : 90, 180, ...

Here

90 contains 6 fifteen times, 9 ten times and 15 six times.

180 contains 6 thirty times, 9 twenty times and 15 twelve times

90 can be divided by 6, by 9, and also by 15.

Here the teacher is advised to think of the activities to be undertaken.

### **Least Common Multiple : L.C.M.**

Lowest Common Multiple is the smallest number which is exactly divisible by each of the two or more given numbers.

Ex Consider 3 and 4.

The multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, 24, 27, ..

The multiples of 4 are : 4, 8, 12, 16, 20, 24, 28,...

The common multiples of 3 and 4 are : 12, 24, 36, ..

The lowest common multiples of 3 and 4 among the common multiples is 12.

Thus 12 is divisible by 3 as well as by 4.

This is the smallest or lowest or least number which is divisible by both numbers 3 and 4

∴ The L.C.M. of 3 and 4 is 12.

Note L C M is always a positive number.

### **Activity :**

Finding the L.C.M of 3 and 4

Teacher distributes the flash cards 1 – 50 among the students and ask them to stand in a circle

- i) The students are asked to raise their *left hand* when asked for the multiples of 4 and show the corresponding flash cards.
- ii) The students are asked to raise their *right hand* when asked for the multiples of 3 and show the corresponding flash cards.
- iii) Teacher asks – multiples of 4? Students rise left hand.  
Teacher asks – multiples of 3 ? Students rise right hand.
- iv) Now the teacher asks that – "those who have raised both left and right hand come two steps forward.
- v) Makes them stand in the order of numbers
- vi) Who is having the smallest number among these ?
- vii) The smallest number represents the L C.M. of 4 and 3.

The common multiples of 3 and 4 are 12, 24, 36, 48.

The least among these is 12  $\therefore$  L.C.M. 3,4 = 12.

Now let us see the methods of finding L.C.M..

### 1. Method 1

The steps to follow are :

- i) Make a list of multiples of each whole number.
- ii) Continue your listing until at least two multiples are common to all the lists.
- iii) Identify the common multiples.
- iv) Choose the least among the common multiples

**Example :** Find the least common multiple of 4, 6 and 8.

Multiples of 4 : 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48.

Multiples of 6 : 6, 12, 18, 24, 30, 36, 42, 48, ..

Multiples of 8 : 8, 16, 24, 32, 40, 48,....

The common multiples of 4, 6 and 8 : 24, 48, 72 . . .

The lowest among 24, 48, 72 is .

$\therefore$  L.C.M. of 4,6,8 = 24.

Disadvantages of this method are :

- i) Listing the multiples of each number is lengthy
- ii) Finding atleast two common multiples to all the lists is time consuming if one number is small and other is larger
- iii) The tendency of committing mistakes is more

### Method 2

The steps to follow are :

- i) List all the multiples of the greater number in a given group of numbers
- ii) Find the smallest multiple which is a multiple of other numbers



**Examples :**

Find the L.C.M. of 6 and 8.

i) The greater number is 8

The multiples of 8 : 8, 16, 24, 32, 40, 48, 56,.....

ii) Finding the smallest multiple of 6 which is also a multiple of 8.

a) Check whether 8 is a multiple of 6 ? No.

b) Check whether 16 is a multiple of 6? No.

c) Check whether 24 is a multiple of 6 ? Yes

∴ L.C.M. of 6 and 8 is 24.

**Example: Find the L.C.M. of 4, 5 and 6.**

The greater number is 6

a) The multiples of 6 : 6, 12, 18, 24, 30, 36, 42, 48, ..

b) i) Check whether 6 is a multiple of 4 and 5 ? No.

ii) Check whether 12 is a multiple of 4 and 5 ? No.

iii) Check whether 18 is a multiple of 4 and 5 ? No.

iv) Check whether 24 is a multiple of 4 and 5 ? No.

v) Check whether 30 is a multiple of 4 and 5 ? Yes

∴ L.C.M. of 4, 5, 6 is 30

The disadvantages of this method are

- i) When more than two numbers are given, checking the lowest common multiples of other (smaller) numbers with the multiples of largernumber, there is more chance of committing mistakes.
- ii) The committing of errors is common
- iii) Too lengthy and time consuming.

**Method 3: By Prime Factorisation Method**

The steps to follow are :

- i) Express the given numbers in terms of the product of *prime factors*.
- ii) Suppose there is a factor occurring more than once, then take that number once and remaining factors which have occurred only once.
- iii) The product of selected numbers is the L.C.M. of the given numbers.

Example : Find the L.C M. of 40 and 36.

Solution: The prime factors of 40 :  $2 \times 2 \times 2 \times 5$

The prime factors of 36 :  $2 \times 2 \times 3 \times 3$ .

The common factors are : 2, 2 (taken once).

The remaining factors are : 2, 3, 3, 5.

$\therefore$  L.C.M. of 40 and 36 is  $= 2 \times 2 \times 2 \times 3 \times 3 \times 5$ .  
 $= 360$ .

$\therefore$  L.C.M 40,36 = 360.

**Method 4:**

The steps to follow are

- i) write down the numbers side by side whose L.C M. has to be found out.
- ii) Go on dividing by a prime number which will divide at least one of the numbers If a number cannot be divided by that prime number, then keep the number as it is
- iii) Repeat the above step until you get the prime number all 1s.
- iv) Product of all the prime divisors used in the L C M.

Example : Find the LCM of 32, 6 and 12.

Solution :

2	6,	12,	32
2	3,	6,	16
2	3,	3,	8
2	3,	3,	4
2	3,	3,	2
3	3,	3,	1
	1,	1,	1

$\therefore$  L.C.M. of 6, 12, 32 is  $2 \times 2 \times 2 \times 2 \times 2 \times 3$

Procedure followed :

- i) Divide 6, 12, 32 by 2 (because all are divisible by 2)
- ii) Divide 6, 16 by 2 keeping 3 as it is.

- iii) Divide 16 by 2, 3 and 3 are kept as they are.
- iv) Divide 8 by 2, 3 and 3 are kept as they are.
- v) Divide 4 by 2, 3 and 3 are kept as they are.
- vi) Divide 2 by 1, 3, and 3 are kept as they are
- vii) Divide 3, 3, by 3.

∴ The product of divisors is L.C.M.

**Problem:**

The circumference of two cylinders are 4 feet and 5 feet respectively. What is the smallest length of wire that can be wrapped round each an exact number of times ?

### **Highest Common Factor (Greatest Common Divisor)**

#### **Concepts**

Factors, common factor, highest common factor, multiples, divisibility or divisible numbers.

**Notations :** Highest common factor : HCF (Greatest Common Divisor gcd).  
We already know factors.

#### **Definitions :**

1. Common factor : A common factor of two or more numbers is a number that divides each of the two or more numbers
2. Highest common factor : The highest common factor of two or more numbers is the largest number, that divides each of the numbers.

#### **Factors**

Consider the example

$12 = 2 \times 6$  ∴ 12 is a multiple of 2 and 6.

$12 = 3 \times 4$  ∴ 12 is a multiple of 3 and 4.

$12 = 1 \times 12$  ∴ 12 is a multiple of 1 and 12.

∴ 2 and 6 are factors of 12.

∴ 3 and 4 are factors of 12.

1 and 12 are factors of 12.

∴ The factors of 12 are 1,2,3,4,6 and 12.

Here in this stage, teacher is advised to give the practice or drill sums on factors (and then arrive at definition) such as the following type :

- $4 \times 5 = 20$  : 4 and 5 are ..... of 20.
- $6 \times 3 = 18$  : the factors of 18 are .....and .. . . .
- Find the factor of : 24, 36, 65, . . .
- 3 is the factor of 12. The other factor is ..... .
- Find all the factors of 42.

Note: Here teacher is advised to see the divisibility rules that are given as a HINT at the END

### Common Factor

Consider the following example :

Let the numbers be 6, 9 and 15.

Factors of 6 : 1,2,3,6

Factors of 9 : 1,3,9

Factors of 15 : 1,3,5,15

The common factor of 6, 9 and 15 is 3

A common factor of two or more numbers is a number that divides each of the numbers

Proof .

$$\begin{array}{r} 2 \\ 3 \overline{) 6} \\ \underline{6} \\ 0 \end{array}$$

$$\begin{array}{r} 3 \\ 3 \overline{) 9} \\ \underline{9} \\ 0 \end{array}$$

$$\begin{array}{r} 5 \\ 3 \overline{) 15} \\ \underline{15} \\ 00 \end{array}$$

Teacher is suggested to give more examples for drilling

### Highest Common Factor (HCF)

There are two methods of finding the H.C.F.

#### Method 1:

- List all the factors of each of the numbers given.
- List the common factors.

iii) Choose the highest among the common factors.

For Example consider 36 and 54

**Solution :**

i) The factors of 36 : 1,2,3,4,6,9,12,18,36

The factors of 54 : 1,2,3,6,9,18,27,54

ii) The common factors are : 1,2,3,6,9,18.

iii) The H.C.F. of 36 and 54 is 18.

The shortcomings of this method are

- i) It is time consuming, because students have to find *all the factors* of each number.
- ii) In case of large numbers the students find it difficult.
- iii) Possibility of committing mistakes is more

### **Divisibility Tests**

The following tests are very useful (for teacher and also students) and worth memorizing.

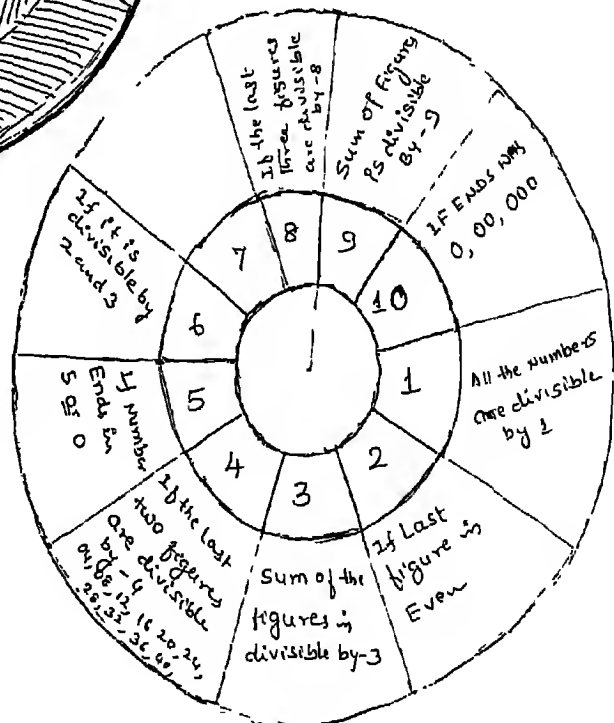
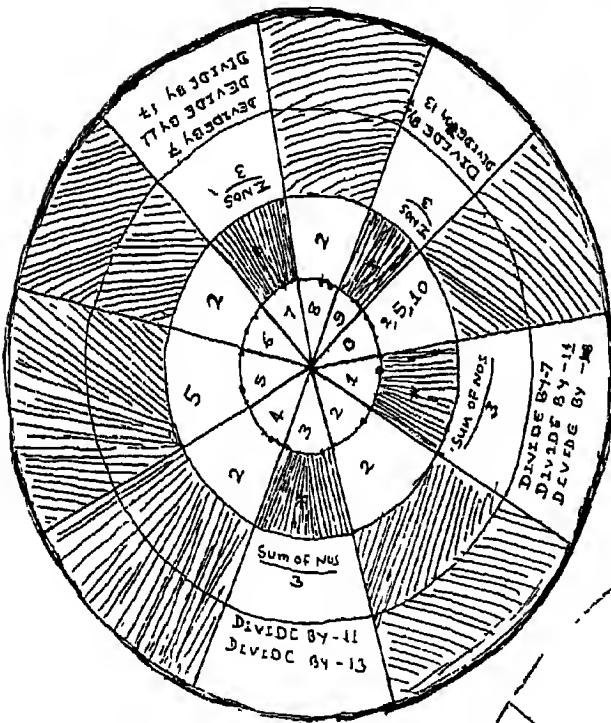
- 1 A number is divisible by 2, if the last digit (i.e. unit place) is even.  
eg 1238, 1564, 1230, 2926, 9652,....
- 2 A number is divisible by 3, if the sum of the figures is divisible by 3.  
eg. 4251 is divisible by 3 because  $4 + 2 + 5 + 1 = 12$  which is divisible by 3.
- 3 A number is divisible by 4, if the number formed by the last two figures is divisible by 4.  
eg 64728 is divisible by 4, because 28 is divisible by 4
- 4 A number is divisible by 5, if it ends in 0 and 5.
- 5 A number is divisible by 6, if it satisfies 1 and 2 above.  
eg. 21354 is divisible by 2 and 3 because  $2 \times 3 = 6$
6. A number is divisible by 7 Take the last digit of a number, double it and subtract the doubled number from the remaining number. If the result is evenly divisible by 7, then the number is divisible by 7. (This may need to be repeated several times.  
eg. 112  $11 - 2 \times 2 = 11 - 4 = 7/7$

- 7 A number is divisible by 8, If the last three figures are divisible by 8.  
(eg. 217584 is divisible by 8 because 584 is divisible by 8).
8. A number is divisible by 9, if the sum of the figures is divisible by 9.  
[eg 8532 is divisible by 9, because  $8 + 5 + 3 + 2 = 18$  which is divisible by 9].
- 9 A number is divisible by 10, 100, 1000. . If it ends in 0, 00, 000, .

#### Example

Tests 1, 3 and 7 above are similar in nature

Tests 2 and 8 are related Also Test 4 can be associated with test 9



**Unit 2 :       Measurements**

Sub-units	2.1	Length, Perimeter and Area
	2.2	Volume, Capacity and Mass
	2.3	Time, Money and Temperature

**Introduction**

"How many fishes does it take (to lay) from Earth to Moon?" - "Just one, if it is long enough".

It is necessary for a teacher to know the importance of the concept – measurement and its utility in our lives. Importance of measurement lies, not only in its vital role in Mathematics, but also in its use in almost every human activity. A child has the intuition by birth to compare as a prelude to measurement. Given two or more things, the child compares the two, to find the bigger, the smaller. As it grows, the child asks questions such as how many? how much? about things he sees around. In different walks of life and variety of locations, measurement comes up. A computerist, a tailor, a tiller, a carpenter, a doctor, an engineer, a music composer, a sportsman, an administrator, an economist – all do measurements of objects of their interest and calculate. They use instrument to measure, procedure and formulas to calculate. The concept of measurement is age old. Geometry – an important part of mathematics is related to measurement of earth (Geo-metrics = earth measure). Since all measurements are expressed as positive numbers (or non negative numbers) with units, the numbers and operations on numbers gain a meaning in practical problems of human interest. Though one is not sure how the first measurement was done, some form of early measurement is believed to have been mentioned in the Rhind Papyrus kept in British museum. Early measurement was taken from the parts of human body and some of these are used even now. Thus measurement of length in terms of Palm, Span, cubit, fathom were got using the fingers and hands. Interest in shadows lead to the invention of first clock about

1500 BC – a device for measuring the shadows. Sundial and water clocks were developed by Romans. It is said Romulus (of Rome) devised a calendar with 304 days in an year. By 46 A.D. Julian Calendar founded after Julius Caesar came into use and is similar to the one used today.

Measurements of different kinds are done for different purposes. For example - A surveyor takes measures of lengths and distances, calculates areas of fields. A scientist calculates volume and surface areas of solids, mass of objects using weighing instruments. A doctor uses his appliances to find the heart beat, pulse rate, blood pressure. He conducts tests to find the sugar level in blood, body temperature – to know the state of health of his patients. A seismologist uses instruments and devices to find the severity of earthquake.

All these reveal how important is the notion of measurement. The unit, as the teacher teaches, unfolds the various types of measurements (spread over different subunits) and their aspects in a sequential, logical and interesting way. The teacher is advised to go through the write ups very carefully and to plan to teach as suggested.

## **2.1 Length, Perimeter and Area**

### **2.1.1 Length**

#### **Concepts :**

- a) Comparison of objects by size - long, small; high, low; tall, short; wide, narrow; longer than, shorter than.
- b) Comparison of lengths using a single improvised unit.
- c) Measuring of lengths using personal units of span, cubit, pace, fathom, etc., in full units.
- d) Knowing and understanding relationships between various units of length.
- e) Convert one unit of length to another.
- f) Measuring length in standard units of length – cm, dcm, m.
- g) Measuring variety of objects by selection of appropriate units of length (cm, dcm and m) after making a sensible estimate.



- h) Expressing the given lengths in the specified unit using relationship between various units.
- i) Solving daily life problems relating to standard units of length.

First and foremost step in the development of concept of length is the comparison of two given objects in terms of their lengths as long, short; thick, thin; near, far, etc. For example, given two pencils to compare, which pencil is longer, which is shorter, or is one as long as the other. A number of activities have been suggested in the class I textbook on pages 130 and 131. These activities may be carried out a number of times over a period of time. In addition, a number of pictorial activities have been given on pages 132 to 143 in the class I textbook. Teachers may convert these activities into more concrete activities, by taking actual objects in place of pictures and make the children carry out these activities before assigning them picture activities like the ones suggested in the textbook. Therefore, we are not going to suggest any additional activities other than the ones given in the textbook for developing the concept of comparison of lengths. Field trips may be arranged to consolidate the understanding.

Next, to make the children learn measurement of length, they should be given sufficient experience in measuring the lengths of objects with the help of other objects which are easily available such as pencils, pens, paper clips, chalk pieces, pencil boxes, or parts of personal body such as span, cubit, palm, fathom, stride, etc. While all measurements are approximate, the use of a variety of different objects such as span, cubit, pace, etc. for measurement, underlines need for a standard unit for measurement such as a meter scale. A meter may be too large a unit for many purposes, so we have to show that smaller standard units such as a ten centimeter or one centimeter length are necessary. The children may be made to discover on their own, the need for smaller units such as decimeter and centimeters. The units should be written in full centimeter, meter, etc. at first and the abbreviations used only when you are sure that

students have fully understood. Abbreviations for measurement units are small letters with no full stops and no 's' for the plural forms i.e. 5 cm, 10m, etc.

Note: Meter – An instrument for measuring. } Oxford Dictionary  
 Metre – Unit of length in SI. }  
 Meter U.S. is the same as metre.

### General Activities

#### 1. Estimation :

Estimation is a very important aspect of measurement as in all number work. The children should be encouraged to state what they think are reasonable estimates of lengths of a given object. They should discuss this estimate with other members and to test it by actual measurement. A recorded estimate should precede an actual measurement. In the measurement of length and width of a classroom for example, the estimated length may be written on a sketch of the shape of the classroom and then the length and width can be measured and recorded. Work like this not only develops the ability of estimation in the children, but also would be the beginning of scale drawing.

2. Measuring of length : Children usually find this activity very enjoying, especially when they can move about the classroom or school to measure on a grand scale. Encourage them to be as accurate as possible in their measurement. When they first use 10 cm scales or meter scales they should use an appropriate number of such scales for measuring the length of an object, which is several ten centimeters long or several meters long to make them understand what they are doing. The idea of repeatedly using a scale to measure length is a more advanced concept and hence should be used only after some time.

3. **Measuring perimeter :** Once the children have mastered measuring lengths, they may be given activities of measuring the perimeters of their notebooks, pencil boxes, desktops, table tops, first measuring bit by bit and then measuring by the use of a string wound around them. They can also find relations among the perimeter and length and breadth whenever the objects are in the form of a rectangle or a square.

Idea of perimeter should be reinforced by measuring large regions such as the perimeter of a hall and playground.

A considerable amount of practical measuring of length and perimeter will be necessary for most children in order to establish fully the concepts being developed. Do not think that the children will be able to master the measurement of length even in a month's time. Only the very able children may be able to do this. Most children need considerably longer time.

### **Preliminary activities**

1, Children should be given the opportunity to have the experience of comparing lengths of common objects such as pencils, straws, strings, ribbons, strips of wood, strides and foot lengths. After this using some of these like pencil or straw as 'standard' lengths, they should measure the lengths of other objects.

2 The children may be encouraged to collect a variety of household containers. Then they may measure the lengths or widths or perimeter of these objects using some 'standard' lengths mentioned above.

Encourage the children to estimate their answers before carrying out the actual measurement.

### **Activities**

Following activities may be carried out by the students in the classroom.

1.
  - i) Measure the length of your desk with your span and record it.
  - ii) Compare your record with the records of your friends sitting on your bench.  
Do all of you have the same answer?
  - iii) Compare your span with your friends'. Are they same?
  - iv) Can you now say why you got different answers for the lengths of the desk when you measured it using your spans?
2. Repeat this activity by measuring the other desks and benches in the classroom, teacher's table, length of the classroom, its breadth, etc. using your span.
3. Repeat this activity by measuring the length and breadth of your classroom with your foot and stride. Compare the records with your friends'. Give reason for different answers.

During all the above activities ask the students first to estimate the measures, record them and then do the actual measurement. Let them then observe how far their estimates were correct.

5. Measure your desk using the length of your notebook
6. Cut a thin stick as long as your span and then measure the length of your classroom using this stick.
7. Cut a piece of string about the size of your stride. Use this string to measure the length and breadth of the classroom.
8. Cut a piece of thin broomstick, which has the length of your stride. Use it to measure the length, width of the playground.

In each of the above activities, children should first estimate their measurements and then measure and compare their estimates.

After every child has performed the activity, let the friends sitting in a bench compare their results with one another. Discuss the reason why they got

same or different answers. Ask them to compare the notebooks, sticks, strings they used to draw the conclusion.

9. Cut a piece of broomstick as long as a half-meter scale. Measure the length, width of the classroom using this stick. Record your answer. Compare it with your friends'. Discuss why almost everyone got the same answer now.

Using the discussions following above activities, the teacher may lead the students to the idea of standard unit of length.

## 2.11 Perimeter

You may introduce the idea of perimeter by the amount of tape or string required to go round the lid of a box or a book. Point out that the perimeter is the length of the boundary. Ask the children how perimeters of various straight edged objects can be found from the length, breadth or lengths of sides. Following types of activities may be undertaken

1. Ask the children to measure the length and width of book nearest to a centimeter. Ask them to add the two lengths and two breadths of the book and record the answer in the notebook

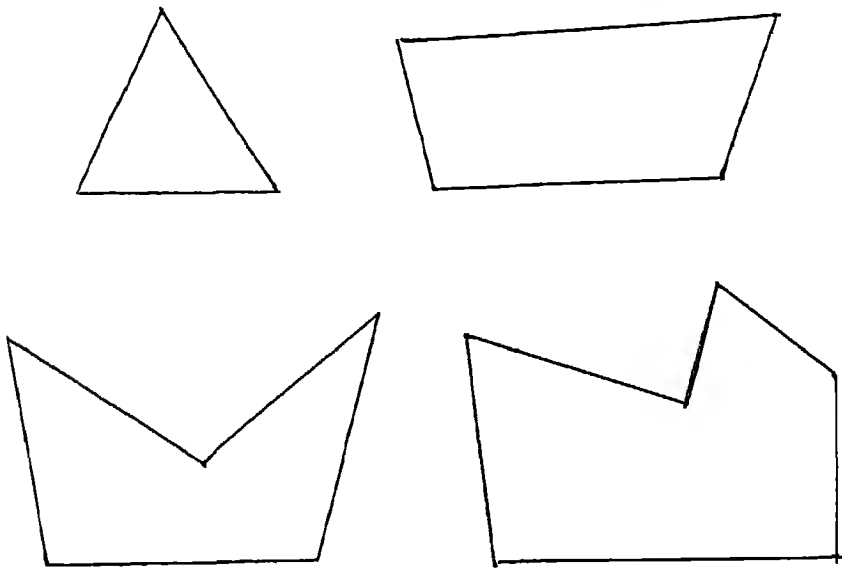
Now ask the children to use a tape or string to wind exactly once round the book length and breadthwise, mark the ends of the tape/string, then unfolding, find the length of the tape/string between the two marked ends. Let them record it and then compare it with the answer recorded earlier. In most of the cases, they will observe that the two records coincide. (Difference, if any, that has crept in between the two records may be explained by the teacher in the

beginning but later left to the children to discuss and find the reason). Let the children repeat this activity for various sized books and enter in the table given below.

	Book 1	Book 2	Book 3	Book 4	Book 5
Method 1					
Method 2					

You may lead them to conclude that the perimeter of the book is the sum of the two lengths and the two breadths.

- 2. Activity 1 may be repeated by taking different lunch boxes, pencil boxes, different rectangular boxes in the school. Each time insist on children recording their measurements.
- 3. Now children may find the lengths of sides and perimeters of different straight edged objects they may find in the school/home and repeat the activity as in Activity 1 and 2.
- 4. Teacher may draw on chart papers the following figures.



Ask the children to measure the lengths of each side of these closed figures and then add them up to find the perimeters. They may also keep string coinciding along the boundaries of these figures, then straighten up the string and find the length and then compare it with the answer obtained by the earlier method.

5. Children may be given activities to measure the length, breadth and then perimeter of their desktop, bench top, table top etc. nearest to a decimeter.
6. Children may be asked to find the perimeter of their classroom, different rooms in the school and of the playground nearest to a meter.

In all these activities, encourage the children to first estimate the answer and then do the actual measurement. This way their ability to estimate will be developed. Also, encourage the children to use a *proper unit* for the measurement. Later they may be asked to convert their answers into different units of length

### 2.13 Conversion of Units

To consolidate the understanding of relationship between different standard units, following group activities may be undertaken by the teacher in the classroom. Each group may consist of 4-5 students. Following are the materials needed for this purpose For each group of students, 10 paper strips of 1 cm each on which 1 cm is written, 10 paper strips of 10 cm each on which 1 decimeter is written, a wooden strip of length 1 meter on which 1 meter is written.

In each group, ask one child to pick up one smallest strip and tell its length. Ask another child to pile up, all strips of length 1 cm each, each time telling 'one centimeter'. Ask another child to pick up another small strip among the remaining strips and tell its length written on it as 1 decimeter. Let another

child pile up all the decimeter strips one over another each time telling '1 decimeter'. Let another child lift the meter strip now and read '1 meter' written on it.

Now, let one child pick a decimeter strip and keep on the table. Ask the children now, "How many 1 cm strips do you think are as long as 1 decimeter strip?" Ask a child to arrange the one centimeter strips along side the decimeter strip and find the answer. Let him count the number of centimeter strips required and say loudly '10, ten 1 centimeter strips make a decimeter'. This 'experiment' may be repeated by all the students in the group. Similarly, let all the children discover that ten 1 decimeter strips are as long as a meter. The activity may be repeated until each child is able to say ten 1 centimeters make a decimeter and ten decimeters make a meter

Activities may now be given to the students to understand the conversion of lengths in one unit to another. In the beginning let them use centimeter strips, decimeter strips and meter strip for the conversion from one unit to another. Later they should be able to do this activity without using paper strips. Activities may be as follows For example to convert 30 cms into decimeters, child will pick out thirty 1 cm strips, arrange ten of them one by the side of the other as a single strip and say 1 decimeter This way they will arrange 3 decimeter strips and say "30 cms is 3 decimeters". Children may repeat this activity a number of times by taking different number of tens of 1 cm strips.

Similar activities may be undertaken to convert decimeters into meters by taking given number of decimeter strips, arrange 10 decimeter strips as 1 meter strip and find out how many meter strips are got. This activity also may be repeated a number of times.



Now the reverse activities may be undertaken by giving the number of decimeter strips to be taken and then find out how many 1 cm strips are needed to make these many number of decimeter strips. Similarly, for conversion of meters into decimeter also activities may be undertaken

As told earlier, after doing the conversion of decimeters into centimeters, using strips, activities should be given to do these conversions without using strips. Teacher may initiate by asking questions like "We are given 30 cms, how many cms are needed to make 1 decimeter?". "How many such 10 cms are there in 30 cms?" "So, how many decimeters are there in 30 cms?" etc Similarly for the conversion of decimeters into cms and so on. These activities also may be repeated a number of times.

### **Problem Solving**

Here, in the measurement of length and perimeter, problem solving should be to make children measure length, breadth and perimeter of their books, pencil boxes, desk, bench, table, chair, classroom, playground, etc in appropriate units of cm, decimeter, meter nearest to the whole numbers.

Further, problems may be given where the lengths of sides of a straight edged closed figure are given and the perimeter has to be found. Similarly, given the perimeter and all but one side, the problem may be to find the missing side. In case of rectangular figures, given the length and breadth problem may be to find the perimeter and conversely given perimeter and either of the length or breadth, the other may be found.

Problem solving should start from concrete problems and then proceed to picture problems and then to oral or word problems. It is very important that the children ultimately are able to do these word problems.

**Tests for self-evaluation**

Suggest one activity each for each of the concepts listed earlier other than the ones given above.

**Subunit 2.2 : Volume, capacity and mass****1. Concepts****2.2.1 Volume/Capacity**

- a) Comparison of sizes of 3 dimensional objects (solids)
- b) Volume/capacity and their measurement
- c) Non standard units
- d) Standard units
- e) Multiples and submultiples of a unit measure
- f) Calculating volumes/capacities

**2. Explanations**

2.21 a) Words such as 'Bigger' and 'smaller' are used when the sizes of two objects (solids) are compared. Accordingly, in this sense, a marble is smaller than a football, the earth is bigger than the moon and a hill is smaller than a mountain. If more objects are given, comparing their sizes one can find the smallest and the biggest among them. They can be listed in the order of their sizes (in the increasing/decreasing order).

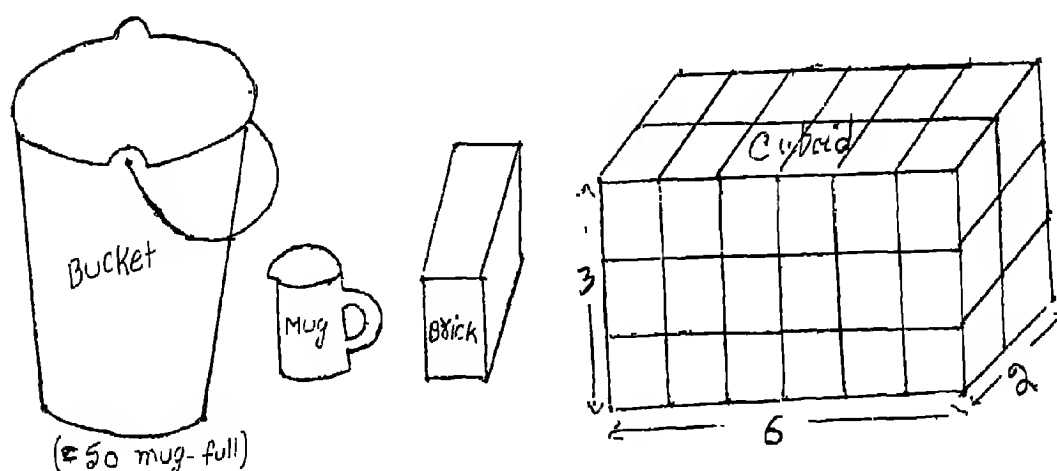
**The words 'capacity' and 'volume' are used in this context.**

**Volume is the amount of space occupied by a solid in space, the capacity is**

**the amount of matter (usually a liquid) which can fill the whole space bounded by a surface.** Then what holds the matter inside it, is the container (hollow unlike solid) like a vessel or a box. Thus we say 'volume of a brick' and 'capacity of a bucket'. However, it is true that these words are used to mean the same thing.

b) Beyond comparison, to get a precise idea of how much is the volume (or the capacity) of a given object, we need a standard object in terms of whose size, the size of the given object is expressed. We call the volume (or capacity) of this standard object as "Standard Unit"

If a bucket holds 50 mugs of water, then the capacity of the mug becomes a unit of capacity and say that the capacity of the bucket is 50 mugs or 50 units. If 36 identical bricks are arranged to form a cuboid, we say that the volume of the cuboid is 36 brick size meaning 36 times the volume of a brick.



c) Capacity of every container cannot be expressed in terms of that of a mug since by a mug, its size is not specific. Likewise, volume of a cuboid cannot be

expressed in terms of that of a brick, a brick whose volume is not specific (it then can be bricks of different sizes) These are examples of non-standard unit of capacity/volume. Therefore, there is a need for a standard unit of capacity/volume in terms of which the capacity of any vessel or volume of any solid can be expressed in a unique way

d) The volume of a cube of length 1 cm is taken as 1 cub. cm (or 1 cc). It is a standard unit of volume The capacity of a container in the shape of a cube of length 1 cm is taken as one milliliter or 1 ml. 1000 ml is called 1 litre and this is taken as a standard unit or capacity

Summing up

A standard unit of volume = 1 cc or 1 cub Cm. and  
A standard unit of capacity = 1 litre or 1l

Note : l means litre                    eg 10 l = 10 litres

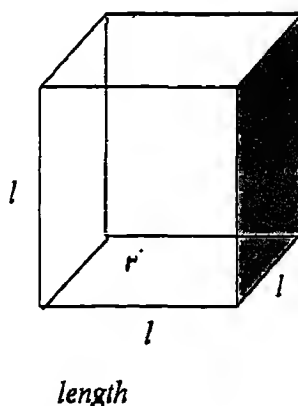
e) Other units of volume are volumes of cubes of edges 1 mm, 1 cm and 1 m. These units are 1 mm cube, 1 cm cube and 1 m cube respectively

Likewise A litre is related to its multiples and submultiples

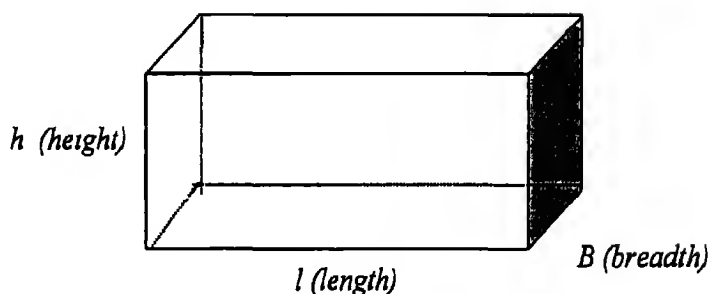
Kilo	1000 l			Multiply each time by 10
		x	10	
Hecto	100l			
		x	10	
Deca	10 l			
		x	10	
<hr/>				
Standard Unit (l)				
		-	10	Divide each time by 10
Deci	1/10 l			
		÷	10	
Centi	1/100 l			
		÷	10	
Milli	1/1000 l			

f) Volume of

- i) A cube = (length of an edge)  $\times$  (length of an edge)  
 $\times$  (length of an edge)  
 $= l \times l \times l$   
 $V = l \times l \times l$  cubic units.



- ii) A cuboid = (length)  $\times$  (breadth)  $\times$  (height)  
 $V = l \times b \times h$  cub. Units



### 3. Learning Activities – Examples and Problems

#### 2.21 Volume and capacity

At the primary level, especially, learning activities are to be designed carefully, keeping in mind, the objectives of learning. A learning activity need not always be a physical activity, though a fairly good number of them are so. Asking correct questions in a proper order with a definite goal in mind, is a good activity to promote learning. Problem solving by the teacher or students is an

important learning activity to ensure correct understanding of the mathematics aimed to be taught.

**Activity 1 :** *Recall* (and list) a number of objects (solids) around, known to the children like – (i) toys of different sizes and sorts, (ii) material objects like – stone, bricks, building blocks, (iii) living beings as human beings, animals, birds, (iv) stationary and moving objects as buildings, vehicles, (v) eatables as – fruits, toffees, dishes we eat daily. Draw their attention to the fact that some are bigger than others in their sizes.

**Activity 2 :** Display pictures/objects of solids/vessels/boxes etc and ask them questions related to comparison of their sizes

**Activity 3 :** Two boxes of different sizes are each filled by marbles of the same size. Children count the marbles in each box. Which box has more marbles? Which one has less marbles? What do you conclude? Which box is bigger/smaller? Can you express the volume of each box in terms of that of the marble ?

**Activity 4 :** Small cuboids of the same size are given to the students. They are asked to construct bigger cuboids using the given ones. Then they are asked the volume of the cuboid in terms of the volume of the cuboids they have used.

**Activity 5 :** A water can is filled using a tea cup – counting the cups of water filling the can. How many cups of water fill the can ? What is the capacity of the can in terms of the capacity of the cup?

**Activity 6 :** Ask such questions as

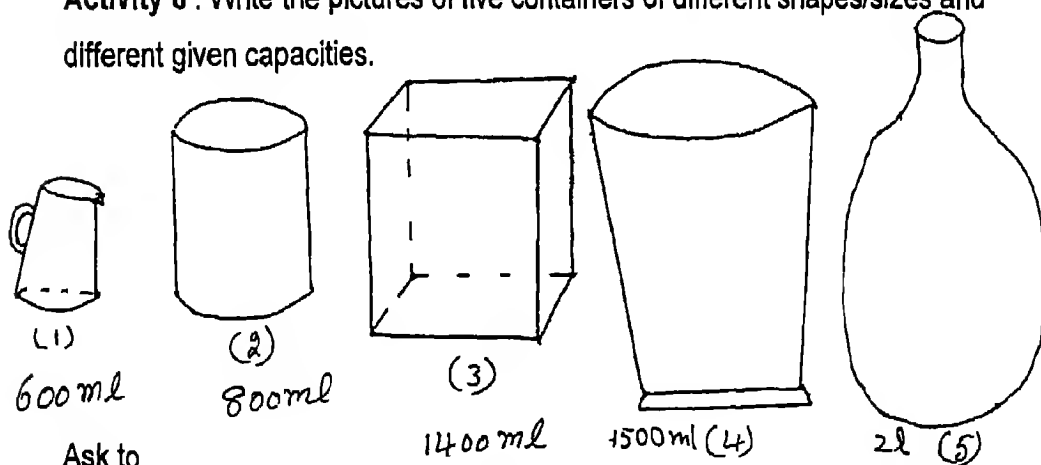
- i) What is the standard unit of (a) volume, (b) capacity?
- ii) Which unit is used to express:

- a) the amount of milk in a bottle, (b) the amount of grains in a bag, (c) the capacity of a can full of oil, (d) the capacity of a tin full of petrol, (e) the amount of a packet full of sugar, (f) the volume of a sugar filled jar.

**Activity 7 :** Oral questions on the following are asked :

- a) The volume of a cuboid shaped tin is 1000 cub. cm. What is the volume of the sugar filling (i) half the tin, (ii) quarter the tin, (iii) three-fourth of the tin.
- b) The amount of oil filling quarter of a can is 250 ml. What is the capacity of the can?

**Activity 8 :** Write the pictures of five containers of different shapes/sizes and different given capacities.



Ask to

- a) arrange the containers in a row in the decreasing order of their capacities.
- b) Identify the containers whose capacity is equal to the sum of the capacities of two other given containers.
- c) If two containers have different shapes, should their capacities be different ?
- d) If two objects have the same volume, should they have the same shape?

**Activity 9 : Constructing questions of the type –**  
**Fill in the blanks:**

a) Capacity

In ml	In l
2000	.....l
1200	.....l
.....	3 l
.....	5/2 l
1500	.....

- i) How many 200 ml make a litre ?
- ii) How many 500 ml make two litres ?
- iii) How many 100 ml make one 500 ml?
- iv) A can whose capacity is 2 l can be filled by using 200 ml full or 500 ml full measuring cups. How many fills does it take in each case ?

**Constructing the problems of the type:**

**Activity 10:**

- a) How many 1 cm cubes are required to build a cuboid of length 10 cm, breadth 8 cm and height 5 cm?
- b) Using all the 1 cm cubes, a cuboid of base 8 cm x 5 cm is built. What is its height ?

From a water tank of capacity 6000 litres containing 5000 litres of water, how much of water must be pumped out to reduce the water in the tank to 3500 litres. How much water must be pumped into the tank now to fill the tank?

A glass jar in the shape of a cuboid has the square base of length 2 cm and height 3 cm. It has water of depth 2 cm. If a solid cube of 1 cm is immersed in the water, by how much the water level in the jar rises?



### Problem Solving Activity

1. A litre oil is put in packets of 200 ml capacity. How many packets are made ?

**Solution :** 1 packet has 200 ml.

$$\therefore (\text{No. of packets}) \times 200 = 1 \text{ l} = 1000 \text{ ml}$$

$$\therefore \text{No. of packets} = 1000/200 = 5$$

There are five packets

2. A cuboid has dimensions  $20 \text{ cm} \times 40 \text{ cm} \times 80 \text{ cm}$  made up of cubes of 1 cub. Cm. Volume. If the cubes are dissembled and a cube is formed, what is the length of the cube so formed ?

**Solution.** Volume of cuboid =  $20 \times 40 \times 80$

$$= 64000 \text{ cub. Cm}$$

$$= 40 \text{ cm} \times 40 \text{ cm} \times 40 \text{ cm}$$

$$\therefore \text{The volume of the cube} = (\text{length}) \times (\text{length}) \times (\text{length})$$

$$= 40 \text{ cm} \times 40 \text{ cm} \times 40 \text{ cm}$$

Hence the length of the cube = 40 cms.

3. 20 litres of milk are distributed into two vessels in the ratio 2:3. How much milk is contained in each vessel ?

**Solution** If the vessels had 2l and 3l, total milk = 5l  $\therefore 4 \times 5 \text{ l} = 20 \text{ l}$  is

distributed as  $4 \times 2 \text{ l}$  and  $4 \times 3 \text{ l}$  i.e. the vessels contain 8l and 12l respectively

4. A wall in the shape of the cuboid is made up of cubes of lengths 15 cms. If the thickness of the wall is 30 cms, length 9 mts, and height 6 mts, find the number of bricks in the wall

$$\text{Volume of each brick} = (15 \times 15 \times 15) \text{ cub. Cm.} \quad (1)$$

$$\text{Volume of the wall} = (30 \times 900 \times 600) \text{ cub. Cm}$$

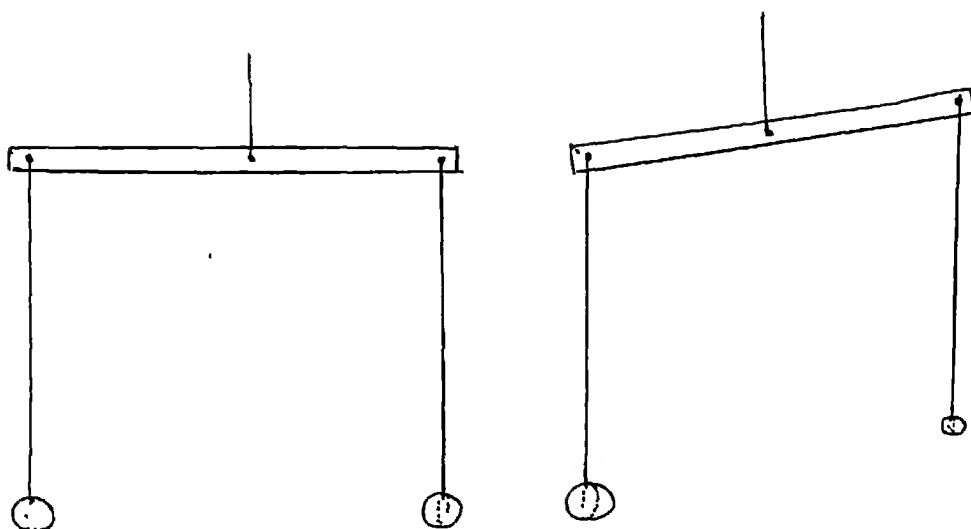
(because 1 mt = 100cms)

$$\therefore \text{No. of bricks} = \frac{\text{Volume of the wall}}{\text{Volume of the brick}} = \frac{30 \times 400 \times 600}{15 \times 15 \times 15}$$

$$\therefore \text{No. of bricks} = 2400$$

### Exercises (for self evaluation) and Enrichment Activity(ies)

1. Design learning activities to teach the following concepts.
  - i) Non-standard unit of volume and standard unit of volume.
  - ii) Multiples and submultiples of litre.
  - iii) Finding the capacity of (a) a cylindrical jar, b) a conical jar, c) a hemispherical bowl.
2. Construct problems on the subunit. Explain stepwise how the problems are solved.
3. In how many ways 32 cubes of length 1 cm each can be assembled to form different cuboids. Tabulate their dimensions.
4. List the pre-knowledge for the first lesson on (a) volume, b) capacity.
5. Explain how volume is different from capacity
6. What are the other units of volume and capacity? Explain.



## Sub Unit : 2.2 Mass

### 1. Concepts:

- a) Comparison of masses of objects.
- b) Non standard and standard unit of mass.
- c) Measurement of mass.
  - i) By using non-standard weights.
  - ii) By using standard weights.
- d) Multiples and sub multiples of unit mass (conversion).
- e) Calculating weight.

### 2. Explanations:

- a) When we hold different objects we feel that some objects are heavier than others. So we need to find the mass of object.
- b) In earlier days mass of an object was measured by using non-standard measures like pebbles, seeds, etc. For example if we measure the mass of a vegetable with small pebbles we may get the mass of a vegetable is 50 pebbles (say), again if we measure the mass of the same with big pebbles we may get its mass as 25 pebbles (say). In order to find the mass of the vegetable accurately we need standard measures.
- c) If we distribute 2kg of rice among 10 persons each gets  $\frac{2}{10}$  kg which is a fraction. If we convert 2 kg into grams then the distribution becomes easy. Therefore, for convenience we need conversion into appropriate units.
- d) Mass and weight are different from each other. Mass is the quantity of matter contained in a body whereas weight of the body is a measure of the force with which gravity pulls it towards the centre at the earth.

#### **Examples/Activities:**

**Activity (a) :** Give different objects to the children and ask them to hold them. The one which exerts more pressure on hand, we call it as heavy and the one which exerts less pressure on our hand we call it as light.

**Activity (b) :** Take any two small containers with different sizes and fill them with sand, lift them separately. Try to find out which is heavier.

**Activity (c):** Collect three stones whose masses are easily distinguishable and arrange them according to their masses.

**Activity (d) :** Prepare a balance using one stick and three thread pieces. Tie two stones of different masses at the end of two thread pieces of equal lengths as shown in the figure below. Ask the students to observe what will happen when we lift the rod using the third piece of thread tied at the middle of the stick. Demonstrate that heavier stone goes down.

**Activity (e) :** Prepare a balance using one stick and three thread pieces. Tie two stones of same mass at the end of the two thread pieces of equal length as in the figure above. Ask the students to observe what will happen when we lift the rod using the third piece of the thread tied at the mid point of the rod. Demonstrate that both the stones rest on a horizontal line. (see p.119)

**Activity (f) :** Divide the children into a number of groups. Provide a balance to each group. Also provide sufficient number of small seeds and sufficient number of big seeds to each group. Now ask each group to find the mass of their pencil in terms of small seeds and big seeds individually. Let them find that the same object have two 'different masses' in terms of the small and big seeds. Explain to them the necessity of standard unit for masses.

**Activity (g) :** A number of simple balances, sandbags, gram masses and various objects (pebbles, seeds, pencils, etc) must be available for the children to use. Ask them to select pairs of objects and by holding them.

In their hands, feel which is heavier, which is lighter. They can use the balance to check their estimates.

**Activity (h) :** Divide the children into groups and provide them balance, sand, weights to each group. Ask them to take out 10 g ; 20 g; 100g; 250 g; 500 g and 1 kg of sand.

**Activity (i) :** Ask them to identify the objects in their surroundings.

- i) that has mass more than 1 kg.
- ii) that has mass less than 1 kg.
- iii) that has mass equal to 1 kg.

**Activity (j) :** Provide a balance and a number of weights of 100g; 200g, 500g; 1 kg. Ask them to keep 1 kg weight in the left pan at the balance and ask them to balance it by keeping 100 g mass in the right pan of the balance. Then by trial and error let them get the following relationships.

- i)  $1 \text{ Kg} = 10 \times 100 \text{ g} = 1000\text{g}$
- ii)  $1 \text{ Kg} = 5 \times 200\text{g} = 1000\text{g}$
- iii)  $1 \text{ Kg} = 2 \times 500\text{g} = 1000\text{g}$

**Activity (k) :** Take the children to the play ground. Divide them into groups, such that each group contains 4 children. Provide a balance and weights to each group. Ask each child to collect 250 g of sand. Now ask each group to pack 1 kg of sand from their collection.

**Activity (l) :** Children should be asked the following questions:

- i) How many grams make 1 Kilogram ?
- ii) How many decagrams make 1 Kilogram?
- iii) How many Hectogram make 1 Kilogram?

- iv) How many milligrams make 1 gram?
- v) How many Centigrams make 1 gram?
- vi) How many decigrams make 1 gram?

Now ask them to make a place value chart of units of mass as given below:

THOUSANDS	HUNDREDS	TENS	ONES	TENTHS	HUNDREDTHS	THOUSANDTHS
1000	100	10	1	1/10	1/100	1/1000
Kilogram	Hectogram	Decagram	Gram	Decigram	Centigram	Milligram

The teacher may find out from the Encyclopedia, how the unit of mass is evolved and explain it to the children

**Activity (m) : Activity to be demonstrated in the Class by the teacher**

By using balance and actual weights find the mass of an iron Ball and then find its mass?

**Problem Solving: (For the Students):**

1. Convert 2023 mg into g, cg and mg.

**Solution:**  $1000\text{ mg} = 1\text{g}$

1000

2023

2

2000

23

..  $2023\text{ mg} = (2023 \div 1000)\text{ g}$

$$= 2\text{g } 23 \text{ mg}$$

$$= 2\text{g } 2\text{cg } 3\text{mg} \quad (10 \text{ mg} = 1\text{cg})$$

2. Convert 5 kg into grams.

$$\text{Solution: } 1000 \text{ g} = 1\text{kg}$$

$$\begin{aligned} \therefore 5 \text{ kg} &= (5 \times 1000) \text{ g} \\ &= 5000 \text{ g} \end{aligned}$$

### Further activities:

**Activity (a) :** Take the children to the play ground, ask them to stand around the see-saw. Allow a stout boy and a lean boy to play see-saw. Ask them to observe. They can observe that stout boy goes down where as a lean boy goes up. They can say that the stout boy is heavier than the lean boy.

**Activity (b) :** Ask the children to make their own standard mass from sand or plasticine. They should eventually compare their own home made masses with actual weights.

**Activity (c) : Estimating mass :** For all balancing activities ensure that the children estimate before actually finding out the mass. Initially encourage them to think in terms of "light" "heavy, but I can hold it", "too heavy for me to hold" and "very very heavy". From time to time ask the children to classify their estimates in tabular form.



**Activity (d) :** Encourage the children to look at the objects in their houses and estimate their masses in terms of some “standard” masses.

**Activity (e) : Mass game :**

Provide 6 containers of same size and shape and labelled A to F. Ask one child to fill each with a different amount of sand, making only slight variations in the mass placed in each container. The lids are then securely sealed. A second child, places the containers in the order of mass from then heaviest to the lowest, estimating first, then using balances only and finally balancing with standard masses. The maximum score is 6 points. If each container is correctly placed in the estimation, the two children can then reverse the roles.

**Activity (f) :** Encourage balancing activities (with recording) in the class, shop and post office.

**Activity (g) : Balancing and weighing :**

Spring scales and balances should be introduced. Give the children plenty of experience of both types. When first using spring scales, the children should weigh objects whose mass they have already found using balance scales.

**Activity (h) : Finding the total mass :**

Give the children four or five objects, for instance groceries each with a different mass. Ask them to place the objects in order from the heaviest to the lightest. Then ask them to place the objects in a large container. Ask "What is the total mass of the groceries"? This activity can be repeated with different sets of objects whose total mass varies from kilograms to tens of grams.

**Activity (i) : Enrichment activity :**

Mass pairs is a game for 2 children. They will need 8 pairs of objects each pair of about equal mass, but not alike in shape. The children display the object in a 4 by 4 array. The first child selects two objects and he estimates whether they are of equal mass. If they are of equal mass, he keeps the pair and selects another pair. If they are not of equal mass he replaces the objects and the other child selects a pair. The game continues until there are no pairs left. The child with the greater number of pairs wins. All estimates should be checked on a balance.

**Exercises:**

1. In which of the following pairs can you definitely say that one is heavier than the other
  - i) flower, tree
  - ii) Chair, table (both are made up of wood)
  - iii) Peacock, Parrot
  - iv) Car, Bus
  - v) Aeroplane, Boat
  - vi) Ant, Monkey

2. (i) If the mass of a rubber ball is make 20 seeds than the mass of iron ball of same size as rubber ball is .....

(more than 20 seeds / less than 20 seeds/ make 20 seeds)

(ii) If the mass of a book is make 50 pebbles then the mass of a pencil is .....

(more than 50 pebbles / less than 50 pebbles)

3. i) Convert 5 kg into grams.

ii) Convert 10 kgs into Hectograms

iii) Convert 3350 miligrams into grams, Centigrams and milligrams.

iv) Convert 8000 g into kg and g

v) Convert 14,660g into kg and g.

vi) Convert 3075 mg into g, cg and mg

**Sub-Unit : 2-3- Time , Money & Temperature****2.3.1-Time****1. Concepts**

1. Day, Week, Month & Year
2. Reading Clock
  - up to hrs.
  - up to half-hrs.
  - up to quarter hrs.
  - Phrases : am, pm , noon, midnight
3. Relations between (i) Secs, Mins, Hrs, Day  
(ii) Day, Week, Month, Year
4. Calendar Reading

**2. Explanations**

- a) Units of time-Day, Week, Month & Year

With the definition of

- (i) a day- The duration in which the earth rotates once about its axis.

(ii) a year-The duration of time taken by the earth to go once around the sun.

- b) Bigger units than day - Week (7 days)  
 - Month (30 or 31 days)  
 - Year ( 12 months or 365 days)

Names of the week days and months.

- C. Smaller Units than day - Hour, Minutes & Second  
 Reading Clock  
 Description of a clock - Dial (face) containing 12 numerals to 12  
 - hands - the hour hand (short) & the minute hand (long)  
 functions of the hands-movement of hands.

(d) Time reading in the clock-

- upto (i) hours-means reading on 1' O Clock, 2' O clock etc  
 (ii) half-hrs-means reading as 1.30, 2 30 etc  
 (iii) Quarter-hrs-means reading as 1.15, 1.45 etc

### Meanings of the Phrases

- (i) 'to' as in 'half an hour to 1' O clock'  
 - means less than 1' O clock by half an hr.  
 (Time 12.30)  
 or as in 'Quarter to 2' O clock'  
 - means less than 2' O clock by Quarter hr.  
 (ie Time 1.45)
- (ii) 'Part' as in 'half past 1' O clock'  
 - means half an hour more than 1' O clock

(Time 1.30)

or as in 'Quarter past 2' O clock

- means Quarter hr. more than 2' O clock

(ie Time=2.15)

(iii) a.m. (ante meridiem) -before noon

p.m. (post meridiem) - after noon

Time between Midnight of a day and Noon is read with a.m. &

Time between Noon and Midnight is read as p.m..

Thus 10 p.m. means 10' O clock before noon.

And 5.p.m. means 5' O clock after noon.

(e) Knowledge of Conversion formulas-

(i) 1 hr = 60 mins

1 min = 60 secs

(ii) 24 hrs = 1 day 12 months = 1 year

7 days = 1 week Also 365 days = 1 Year

(30 days) = 1 month & 366 days = 1 leap Year

or

(31 days)

(f)

(i) Description of a calendar and its use

that reading a calendar, one knows the day, date, the month and the year.

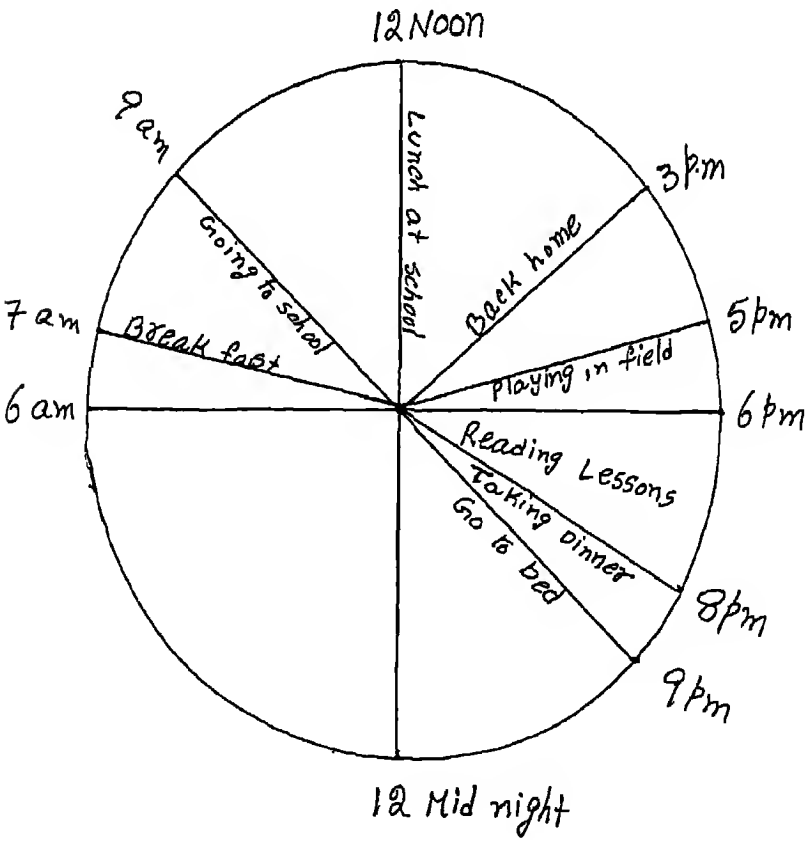
(ii) Different calendars with special emphasis on National Calendar

(iii) When does the date change? (The date changes at midnight).

3. Learning Activities

Activity 1: Recalling the activities of a child, when and which activity at what time

Draw a circular chart to indicate the various activities-preferably with appropriate pictures pasted in approximate places.  
Ask Children to tell the activities they do in their order from morning to night.



Activity 2: Display a clock. Describe it-Variou parts, that the children see

- what is it used for?

Ask the children to-prepare a clock-model

- identify the numerals on the face, how many and list them.
- Identify the hands of a clock.
- The purpose of each hand.

Show the working clock.

ASK- Which hand is faster and which one slower Setting the hour hand to different numerals (hrs) let the children read the time upto hours Ask them to write pictures of a clock showing the given time, writing the hands at correct positions.

Like- write the pictures of a clock showing

- (i) 12 ' O clock                      (ii) 2' O clock                      (iii) 6' O clock  
(iv) 10' O clock.

Extend this activity to develop the ability to read to half hrs, to quarters.

### **Activity 3: Ask the children to answer the following questions**

1. What is the time taken by the minute hand to go round once?
2. In one hour, how far the hour hand moves?
3. How much time is taken by the hour hand to go round once?
4. When will the two hands coincide?
5. What will in the position of the minutes hand when the hour hand shows 6' O clock.
6. How many times the same time is shown by a clock in a day?

### **Activity 4**

Ask the children to bring one sheet ( ie one month) of an old calendar. Ask them to find the answers to the following questions



- (i) What is the year of the month?
- (ii) What is the name of the month?
- (iii) How many days are there in that month?
- (iv) How many each weekday are there in that month?

### Activity 5

Let the children bring a calendar. Let them find

- (i) The month having 28 or 29 days.
- (ii) The months having 30 days.
- (iii) The months having 31 days.

Let them Sing the following song in chorus-

Thirty days have September  
April, June and November  
All the rest have thirty one  
Except February alone  
Which has twenty eight days clear  
And twenty nine days in a Leap year

Put the following questions to the children-

- 1 How many days are there in an ordinary year?
- 2 How many days are there in a leap year?
- 3 How many weeks are there in an year?
4. How many months are there in an year?
- 5 How many weeks are there in a month?

**Activity 6**

Let the students fill in the blanks in the following statements.

1. A day has ----- hrs.
2. Duration of 24 hrs is called a -----.
3. A week has -----days
4. First day of a week is Monday The last day of the week is -----
5. In one year there are 12 -----.
6. In an year August. 15 fell on Sunday. August 31 in that year was-----  
-----.
7. There may be atmost -----days in an year
8. Yesterday it was Sunday. Day after tomorrow it is -----
9. The no of days from October 2<sup>nd</sup> of an year to the next January 26<sup>th</sup> is -----  
-----, if the year and the next are ordinary years.

**Activity 7**

Ask the children to find the months in which the following days fall-

- (a) Your birthday
- (b) Republic Day
- (c) Children's Day
- (d) Teacher's Day
- (e) Mahatma Gandhi's Jayanthi
- (f) Independence Day.

**Solving Problems**

1. Now it is quarter to 3' O clock After how many hrs and mins it is half past 7' O clock

Solution: It is quarter to 3' O clock, now

Therefore now the time is 2 45

After wards, the time is half past 7' O clock

Therefore the time then is 7.30

The duration between the two instants

$$\begin{aligned} &= 7.30 - 2.45 \\ &= \boxed{4 \text{ hr } 45 \text{ mins}} \end{aligned}$$

2. Using the phrases a.m. / p.m. express the time at the moment described below.

5 hrs 15 mins

- a) after half past six p.m.
- b) before 6 p.m.
- c) after quarter to 9 p.m

Solution:

$$\begin{aligned} \text{a) Given time} &= \text{Half past six a.m.} = 6.30 \text{ a.m.} \\ &+ 5.15 \\ \hline 5 \text{ hrs } 15 \text{ min after} &= 11.45 \text{ a.m.} \\ \hline \end{aligned}$$

$$\begin{aligned} \text{b) Given Time} &= 6 \text{ p.m.} \\ &- 5.15 \text{ hrs} \\ \hline 5 \text{ hrs } 15 \text{ mins before} &= 12.45 \text{ p.m.} \\ \hline \end{aligned}$$

$$\begin{aligned} \text{c) Given time} &= \text{quarter to 9. p.m.} = 8.45 \text{ p.m.} \\ &+ 5.15 \text{ hrs} \\ \hline \end{aligned}$$

5 hrs 15 mins after quarter 9.p.m. = 2.00 a.m.

3. A train leaves Bangalore at half past 9 pm and reaches Hyderabad at quarter to 6 am. What is the Journey time?

**Solution:** Leaving time : 9.30 p.m.

Reaching time : 5.45 a.m.

(i) Duration from 9 30 p.m. to 12 midnight = 12.00  
 - 9 30  
 -----  
 = 2 hrs 30 mins (i)

$$\begin{array}{rcl} \text{(ii)} & \text{Duration from 12 midnight to 5.45} & = 0 \text{ hrs} \\ & & + 5.45 \\ & & \hline & & = 5 \text{ hrs } 45 \text{ mins } \quad (2) \end{array}$$

$$\begin{aligned} \text{Duration of Journey} &= (1) + (2) = 2 \text{ hrs } 30 \text{ mins} \\ &\quad + 5 \text{ hrs } 45 \text{ mins} \\ &= 8 \text{ hrs } 15 \text{ mins} \end{aligned}$$

4. A ship leaves Cochin harbour at 6 am on Monday and reaches the Lakshadweep island capital at 2 p.m. on Wednesday of the same week. Find the travel time taken by the ship.

**Duration from 6 am of Monday to 12 Noon**

$$\begin{array}{rcl}
 & = & 12.00 \\
 & - & 6.00 \\
 & = & 6 \text{ hrs } (1)
 \end{array}$$

$$\begin{array}{rcl}
 12.00 \text{ Noon of Monday to Midnight} & = & 12 \text{ hrs } (2) \\
 \text{Midnight} & = & 0 \text{ hr. of Tuesday} \\
 0 \text{ hr of Tuesday to } 0 \text{ hr of Wednesday} & = & 24 \text{ hrs } (3) \\
 0 \text{ hr of Wednesday to } 12 \text{ noon of Wednesday} & = & 12 \text{ hrs } (4) \\
 12 \text{ noon Wednesday to } 2 \text{ p m. (on Wednesday)} & = & 2 \text{ hrs } (5) \\
 \text{Hence the travel time} & = & (1) + (2) + (3) + (4) + (5) \\
 & = & 6 \text{ hrs} + 12 \text{ hrs} + 24 \text{ hrs} + 12 \text{ hrs} + 2 \text{ hrs}
 \end{array}$$

<b>= 56 hrs</b>
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#### 4. Exercises (for self-learning) & Enrichment activities

##### 1 Design learning activity for teaching each of

- (a) Hour as unit of time and in such multiple– minutes & Second
- (b) Reading the clock. Both conventional Clock /Watch & Digital Watch
- (c) Calendar Reading

##### 2. Collect information about the history of calendar.

##### 3 Write a brief note on 'National Calendar'

## 2.32: Money

- 1. Concepts:**
- 1. Indian coins and currency notes of different denominations(values)
  - 2. Paise-Rupee relation  
(Conversion formula) and notation for writing the amount
  - 3 Money transactions involving number operations –Buying & Selling

#### 4.Profit & Loss –Business Problems.

### 2.Explanations

1. Thro' Money we get what we want Using money we exchange things
2. Formation of money of a given value using coins/ currency notes – different combinations.
3. For money transactions(business) we need coins and currency notes of different values.
4. Decimal notation is used to represent the money of given value. For example, an amount of Twelve Rupees and Thirty Paise is written as Rs.12 30. One also may use the notation

Rs	Ps
12	30

- 5 A given amount can be expressed in paise alone or in terms of Rs. and paise
6. The price at which a Merchant purchases an article is called the cost price of the article or C.P. The price at which the merchant sells the article is called the Selling Price or S.P. of the article.
  - (a) If S.P. is higher than C.P. then S.P.-C.P.is the profit for the merchant .
  - (b) If C.P.us higher than S.P, then C.P.-S.P is the loss for the merchant

### 3. Learning Activities, Examples & Problems

**Activity.1.** Recalling the various types of coins and currency notes (a) orally (b)by showing their pictures (c) the coins and currency notes

Asking the children to recognize/differentiate /compare the values of /discriminate the coins/notes of different values.

**Activity 2** Making dummy coins/ currency notes of different values. Framing different amounts by different coins/ notes (of different values)

Thus two rupees can be formed by any of the following combinations

- (i)200 paise coins
- (ii)Hundred 2 paise coins
- (iii)Forty 5 –paise coins
- (iv)Twenty 10 paise coins

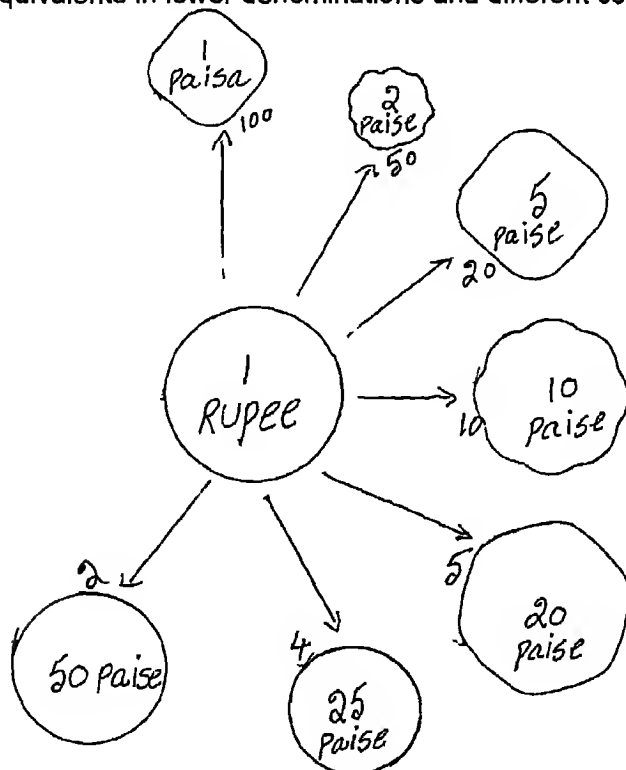
- (v) Ten 20 paise coins
- (vi) Eight 25 paise coins
- (vii) Four 50 paise coins
- (viii) Two 1 Rs.coins/ notes
- (ix) One 2 Rs.coins/notes

Solving simple problems based on the ideas –

eg:- Raju gave Rs.5 to Ram in 2 Rs.coins, 1 Rs coins, 50 Ps coins and 25 ps.coins. If the amount had two 1 Rs.coins, how many coins of each other type was there.

**Activity 3** Initiating plays in which cards and paper cuttings of different sizes and shapes (resembling the actual coins/ currency notes, to some extent) are used to promote the understanding of (a) formation of given amounts (b) counting coins and currency notes given and calculating the amount (e) buying articles(which are dummies like toys, books, toffees, food items, playthings, house hold articles etc ,)

**Activity 4** (a) Preparation of charts containing picture of coins/currency notes and their equivalents in lower denominations and different combinations



(b) Given an amount expressing the amount as combinations of coins/notes of different values(lower) through pictures.

eg.      5 Rs.       $\longrightarrow$  2 Rs. Coin + 2 Rs. Coin + 1 Rs. Coin  
Note

### Activity 5 (Problem solving)

**Asking the children to fill-in the blanks in the following statement**

1. We write Rs..... for ten rupees forty paise.
2. Rs.13.25 means . .... Rupees ..... paise

Sl. No.	Article No	Cost (Per piece)	Money paid	Money to be received
a)	Toy (1)	Rs.4.50	Rs.5.00	50 Ps
b)	Note book(10)	-	Rs.10 00	Rs. 2 40
c)	Pencil (1)	Rs.3-00	Rs 15.00	Rs.——
d)	Eraser(2)	Rs.1-00	Rs.2.00	Rs.——
e)	Ball( )	Rs.6-00	Rs.10.00	Rs.4-00
f)	Biscuits(5)	——	Rs 20.00	Rs.2-50



4.Tick(✓) the correct word

SL. NO.	COST PRICE	SELLING PRICE	PROFIT	LOSS	NONE	REASON
1	Rs.16-00	Rs.18-00	✓	-	-	SP > C P
2	Rs.25-00	Rs.30-00				
3	Rs.40-00	Rs.40-00				
4	Rs.10-00	Rs.12-50				
5	Rs.30-50	Rs.30-00				

Activity 6: Prepare a fairly large No of coins and currency notes of different denominations, using card cut outs and papers. Collect different articles as dolls, Pencils, toffees etc., with price tags. Ask a student to play, the seller, and some others play ' the customers'. Let them play buying /selling.

Customer paying the exact amount for the article bought.

Activity 7 Ask children to prepare the following charts.

How many coins for each amount?

Money (Amount)	50 Ps.	No.of coins		10Ps.	5Ps	3Ps.	2Ps.	1Ps.
		25Ps	20 Ps					
1Rs								
2Rs. 50Ps								
4Rs 25Ps								
6Rs.								
8Rs								
10Rs.								

Activity8: Ask oral questions(without paperwork) like

- 1) How many 50Ps.coins make 10Rs.?
- 2) A 5Rs.coin, a 2 Rs.coin, a 1 Rs.coin, a 50 Ps coin and a 25 Ps coin amounts to how much of money
- 3) Change for Rs.10 consisted 4 coins. What are the denominations of the coins and how many of each type.

- 4) I have two coins amounting to 7Rs. If I spend the coin of lower value I am left with 5Rs. what are the values of the coins I have.
- 5) What combinations of coins. Whose value is not less than 1Rs. make Rs.5?

### Activity 9: (Problem solving)

(1) I went to the market with Rs.25 on the way some one who owed me Rs.15 paid me. The expenditure was as follows:

Item	Amount spent
	Rs.Ps.
Vegetable	8.50
Fruit	5.25
Flower	4.00
Sweets	6.00
Bus charges	3.50

Solution: Total amount I had  
= Rs 25+Rs.15=Rs.40(1)

Total Expenditure= 8.50  
5.25  
4 00  
6.00  
3.50

=27 -25 (2)

How much had I when I reached home?

Balance = (1)-(2)  
40-00 -27 25  
=12Rs.75Ps.

**Ans: I had Rs. 12 and Ps.75 on reaching home**

2 Prepare the bill for the following purchaser.

Sl No.	Item	Cost Piece	No.of pieces bought	Amount	
		Rs.Ps.		Rs.	Ps.
1	Note Book	6.50	4		
2	Ball Pen	4.00	2		
3	Easer	1.00	3		
4	Ink bottle	3.00	1		
5	Pencil	2 50	6		
6	White sheet	0.50	20		

Solution	Last column	Rs.Ps.
1	$4 \times 6.50 =$	26.00
2	$2 \times 4.00 =$	8 00
3	$3 \times 1.00 =$	3 00
4	$1 \times 3.00 =$	3.00
5	$6 \times 2.50 =$	15.00
6	$20 \times 0.50 =$	10.00

3. Some school children and 5 teachers went to an exhibition on children day. The entry fee for each child is Re 1 and for an adult Rs 5. The total entry fee paid was Rs.50. How many children went there?

**Solution.** Total entry fee: = Rs 50=00  
 Fee of 5 teachers  
 @ Rs.5/- =Rs. 25=00

$\therefore$  The entry fee for children @ Rs.1/-  
 Hence there were 25 children

3. A cloth merchant sold cloth for a profit of Rs.400. If the profit per meter of cloth was Rs 2, how many metres of cloth did he sell?

**Solution** Profit per mt = Rs 2  
 Total profit =Rs 400

$\therefore$  (length of the cloth sold)  $\times$  (profit per meter) = (Total profit)

or (length of the cloth sold)  $\times 2 = 400$

$$\therefore \text{length of the cloth sold} = \frac{400}{2} = 200m$$

5. A rice merchant sells an amount of rice at Rs.16 per Kg. making a profit. Had he sold the amount of rice for Rs.12 per Kg. he would have incurred loss. If the profit : loss = 1:3 find the cost price of the price per Kg.

**Solution** Suppose he sells 1 Kg. of rice  
 Then the profit =  $16 - (C P)$   
 Loss =  $(C P) - 12$

$$\text{Since } \frac{\text{profit}}{\text{loss}} = \frac{1}{3}, \quad \frac{16 - CP}{CP - 12} = \frac{1}{3}$$

$$3 \times 16 - 3(C.P.) = (C.P.) - 12$$

$$48 + 12 = 4(C.P.)$$

$$\therefore C.P. = \frac{60}{4} = \text{Rs. } 15 \text{ per Kg.}$$

**$\therefore$  Cost Price per Kg.  
=Rs.15**

#### 4. Exercises (for self evaluation) and enrichment activities

1. Prepare a chart depicting the coins and currencies of the following countries – (a) USA (b) UK (c) Russia (d) China & (e) Japan
2. Design learning activities to teach each of the following-
  - (a) Need to use coins/Currencies
  - (b) Different coins/ currency notes used in our country
  - (c) Profit
  - (d) Loss
3. Design play-way method to teach –
  - (a) Buying and selling
  - (b) Money exchanges (changing one combination of coins and currencies to another)
4. Explain how people transacted in the absence of money in the form of coins and currency notes (Hint: Barter system)

### 2.33 Temperature

The lesson on temperature (Let's learn Mathematics –book five) is fairly expansive and taken the needs of a teacher who wants to teach this to pic therefore we do not intend to elaborate on the different aspect of the content and teaching methodology

The concept temperature is familiar to all including the children of the present day. They are exposed to media (printed and video) –weather bulletins, daily temperature in different parts of the country.

As all know, there are two system of measuring and expressing temperature- celcius and Fahrenheit denoted by  $^{\circ}\text{C}$  and  $^{\circ}\text{F}$ .

- (1) The teacher who teaches, gives instances of cool/hot bodies-the need to know the extent of cold and heat. Compare the two.
- (2) Next, measuring temperature of a body-Different system. The pictures the two thermometers-one on celcius scale and the other on Fahrenheit scale. Explain the construction of thermometer reading the thermometer The conversion table and the techniques for conversion from one to the other. In a F scale there are 180 divisions  $(212 - 32) = 180$ . In a C scale there are 100 divisions.  
 $100 : 180 = 5 : 9$

So,

$$(1) \quad ^{\circ}\text{C} \times \frac{9}{5} + 32 = ^{\circ}\text{F}$$

$$(2) \quad \begin{aligned} (^{\circ}\text{F} - 32) \times \frac{5}{9} &= ^{\circ}\text{C} \\ 0^{\circ}\text{C} (\text{Freezing point}) \\ &= 32^{\circ}\text{F} \\ 100^{\circ}\text{C} (\text{Boiling point}) \\ &= 212^{\circ}\text{F} \end{aligned}$$

An explanation about

- a Clinical thermometer- its construction
- who uses and for what propose?
- Reading a clinical thermometer

Normal body temperature in  $^{\circ}\text{C}$  is  $37^{\circ}\text{C}$  and in  $^{\circ}\text{F}$  is  $98.6^{\circ}\text{F}$

The range of values of body temperature (Normal)

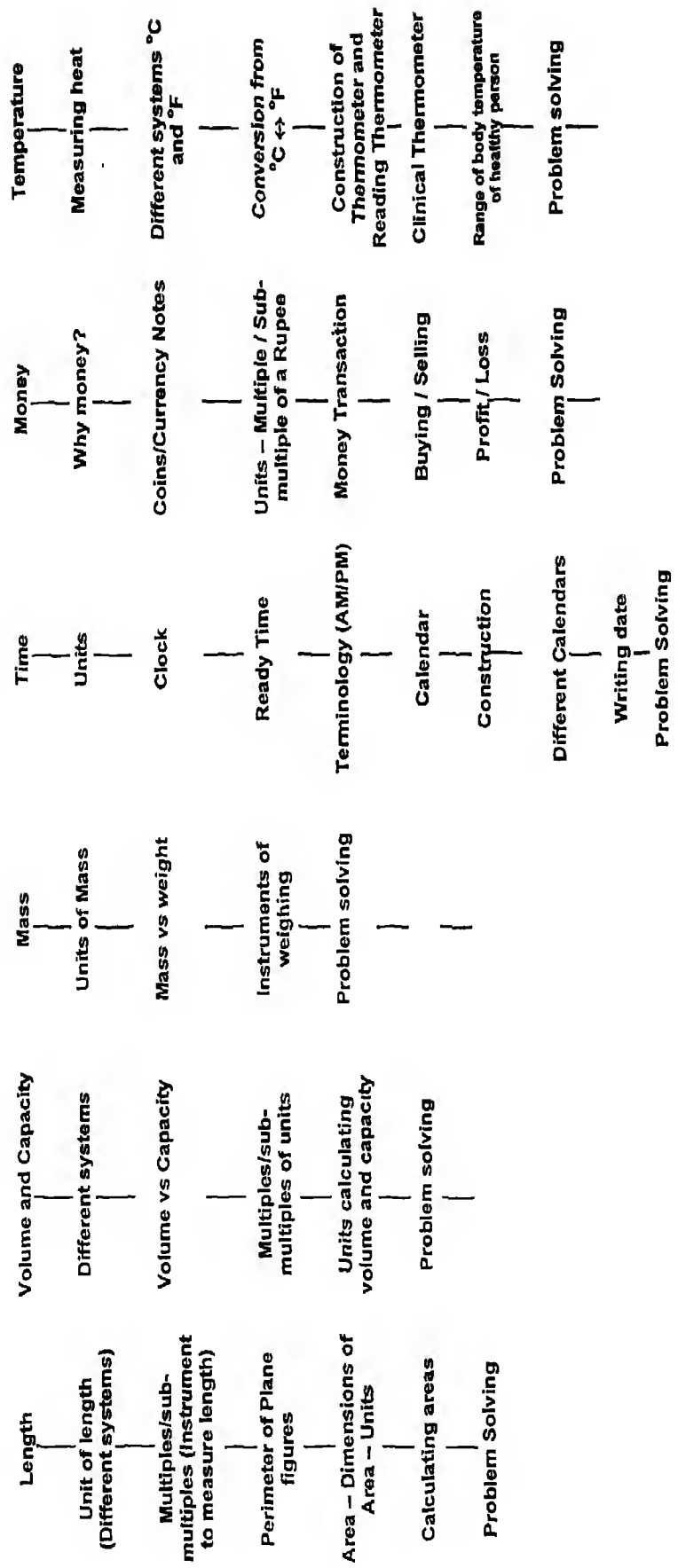
### Exercises (for self-evaluation) and enrichment

- (1) Design learning activities to teach-
  - (a) Measuring temperature
  - (b) Two systems of temperature and their conversion from one to the other.
  - (c) Using clinical thermometer

2. Arrange a meet of the children with a medical doctor so that the children know more about body temperature and normal health.

3 Answer all the questions found in the text book (Let's learn Mathematics-Book Five)

Measurement (An Overview)

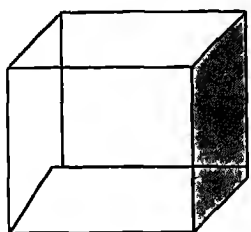


## Unit 3

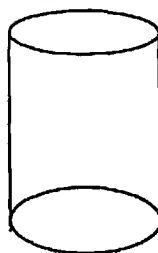
### Geometrical Shapes

#### Introduction:

You have seen tables, books, papers, boxes etc. We recognize them by the shape of their surface. These shapes have particular properties. The shape of the surface of a tea packet is different from that of a 2 kg tin of oil. We study the properties of these shapes in geometry.



Tea Packet



2 kg oil tin

The word "Geometry" is derived from the Latin word "Geo+Metrea" i.e. earth measure. Very similarly sounding Sanskrit equivalent word also surprisingly means the same- Jya (earth) + Mithi (measuring). In the initial stages of civilization, the subject was developed to serve only the practical need of the earth measurement. Excavation of Harappa, Mohanjodaro and Ujjain reveal that all these cities were laid out in a very planned way. This only shows that ancient Hindu Mathematicians were quite proficient in "applied knowledge of this subject". The work contained in Sulvasutra's Brahmagupta formula of the area of a cyclic quadrilateral, a close approximation of the value of  $\pi$  reveals that in ancient India the study of geometry was pursued quite seriously.

Prof C N Srinivasa Iyengar, the noted Indian Mathematician wrote about Sulva-sutra in his book "The History of Ancient Indian Mathematics." "The Sulvas explain a large number of simple geometrical constructions like Construction of squares, rectangles, parallelograms, and trapezia. These and others involve the following theorems

- 1) The diagonal of a rectangle divides it into two equal halves.
- 2) The diagonal of a rectangle bisect each other and the opposite areas are equal.
- 3) The perpendicular through the vertex of an isosceles triangle on the base divides the triangle into two equal halves.
- 4) A rectangle and a parallelogram on the same base and between the same parallels are equal in area
- 5) The diagonals of a rhombus bisect each other at right angles.
- 6) The Pythagoras Theorem.
- 7) Properties of similar rectilinear figures".

A considerable number of geometrical facts were known as early as 1700 B.C. to Greeks, Babylonians and Hindu Mathematicians. The theorem named after famous Greek Mathematician Pythagoras is as follows

"The square on the hypotenuse of a right angled triangle is equal to the sum of the squares on the other two sides".

This theorem was used by Babylonians, Hindus and Egyptians in their land measurements long before Euclid and other Greek Mathematicians

Euclid's work "Elements" is one of the greatest and most elegant mathematical work around 300 B.C. His unique contribution lays in organizing almost all the isolated geometrical facts at his time into a logically developed system. His famous work "Elements" does not contain practical geometry of measurement, but only strictly logical development of plane geometry and solid geometry. All theorems are proved from a small number of definitions and postulates. Under the influence of Greeks geometry developed as a deductive system and remained so, for about 2000 years.

Recent development in the field of geometry include many imaginative ideas which are new in content and approach. In an attempt to prove Euclid's fifth postulate, new non-euclidian geometries have been created. All these new geometrics have found immense use.



This unit has been divided into four sub-units. The first sub unit gives activities and situation illustrating the different concepts like top, bottom etc. It also gives activities regarding geometrical shapes like cuboid, triangle, rectangle and circles.

The second sub-unit deals with activities involving straight and curved lines and surfaces. Here the students are introduced to paper-folding exercises for making shapes of triangles, squares etc

In the third Sub-unit, the concepts of ray, plane and angle have been explained with the help of numerous examples and geometrical situations.

In the last Sub-unit, certain easy constructions and patterns including angle and circles have been given. The definitions of perpendicular and parallel lines are given here. Certain elementary facts about triangle and circles are also verified here.

**SUB- UNIT No.3.1: Basic ideas of Geometrical shapes**

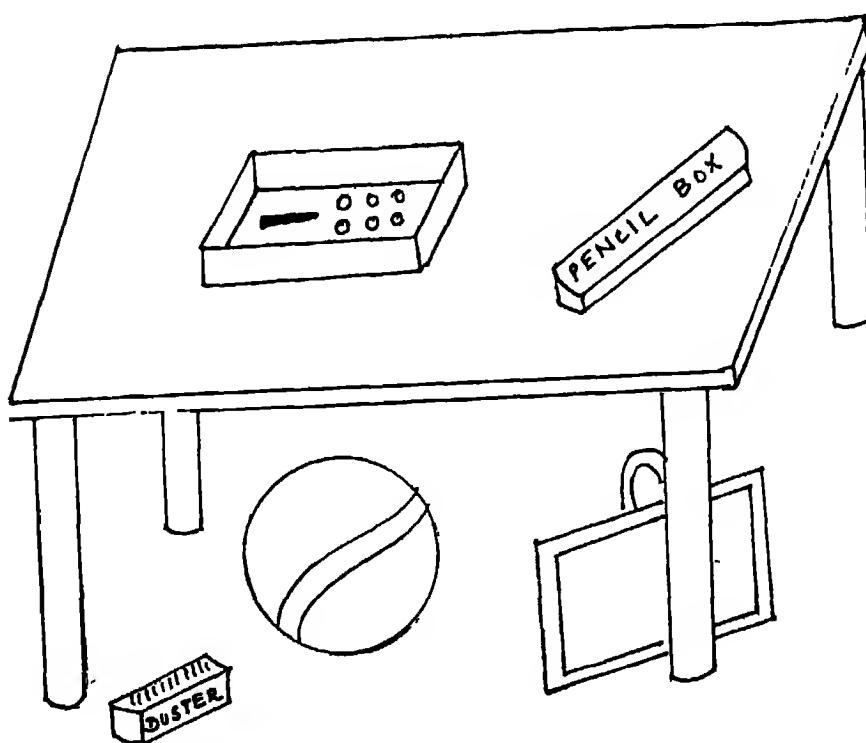
- 3.1-1: Develops and uses vocabulary of spatial relationships (top, bottom over under, inside).
- 3.1-2: Recognizes basic shapes such as cubes, cuboids, sphere, cone, cylinder, triangular prism etc.
- 3.1-3: Makes straight lines by following straight edged objects, stretched strings and with a ruler
- 3.1-4: Draws horizontal, vertical and slant lines.
- 3.1-5: Draws (free hand) shapes of square
- 3.1-6: Draws shapes of triangles squares, rectangles, circles, half-circles and quarter –circles, by paper following and paper cutting
- 3.1-7 Recognizes curved lines and straight lines, closed and open figures, flat and curve surfaces.
- 3.1-8: Makes simple patterns and models out of given shapes
- 3.1-9. Discovers and narrates simple characteristics of shapes.

### Sub Unit 3.1: Basic Ideas of geometrical Shapes.

#### 3.1-1 Develops and uses vocabulary of spatial relationships (top, bottom, over, under, inside, outside etc.,)

**Explanation/Activity:** The teacher could develop this concept in a very lively way with students' participation as follows.

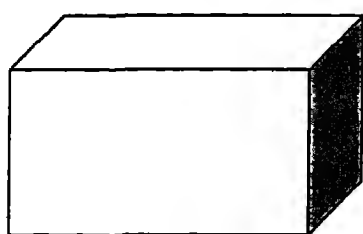
- i) Place a table at the center of the classroom and keep a few things like a pencil box, a box, a chalk piece, a notebook over it and a few things like a ball, a duster, a bag below it. With the help of these things introduce the mentioned words mentioned above as follows.



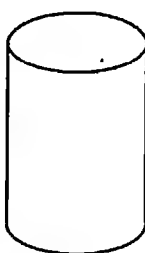
- i) The pencil box is on the table;
- ii) The ball is under the table
- iii) The piece of chalk is inside the box.

**3.1-2 Recognizes basic shapes such as cube, sphere, cuboid, cone, cylinder, and rectangular prism.**

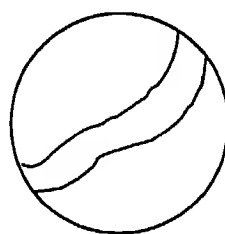
**Explanation:** Models of various geometrical shapes should be presented to the children. The children themselves should handle and explore these shapes as much as possible.



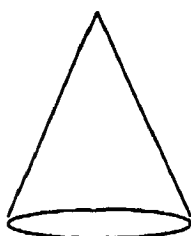
Cuboid



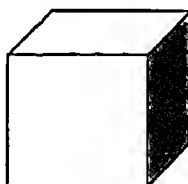
Cylinder



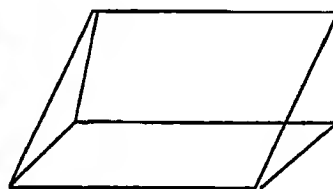
Sphere



Cone



Cube



Triangular Prism

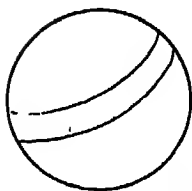
The children should be made to realize that some of these solids have flat surfaces and some have curved surfaces

**Activities:**

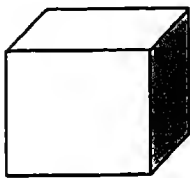
- 1) Keep a few solids like cuboid, cylinder, cone, sphere, cube on the table and call the students to the table. Ask them to group the above solids as having
  - (i) only flat surfaces
  - (ii) only curved surfaces.
  - (iii) both flat and curved surfaces.
- 2) Using soap cakes and clay encourage the children to make solids having

- (i) flat surfaces only
- (ii) curved surfaces only.

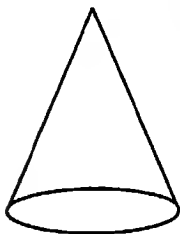
3) Use a matching activity to make the children learn the names of the basic solids



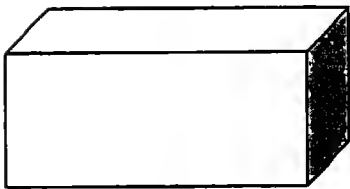
Cuboid



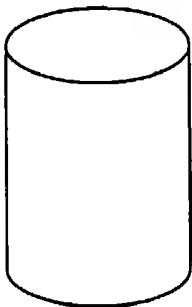
Cone



Cylinder



Cube



Sphere

Ask the children to bring a few solids from their home and name them according to their surfaces.

- Evaluation:-**
- (i) Name three spherical objects
  - (ii) Name two conical objects
  - (iii) Name three cylindrical objects
  - (iv) Name two objects which do not have any flat surfaces
  - (v) Name three objects which have only flat surfaces.

### **Enrichment Activity (For teachers)**

- (i) Excluding spheres and balls from your daily life give two examples of solids which have only curved surface.
- (ii) Build a cuboid without using a cube.

### **3.1-3 Makes straight lines by paper folding, straight edged objects, stretched strings, and with a ruler.**

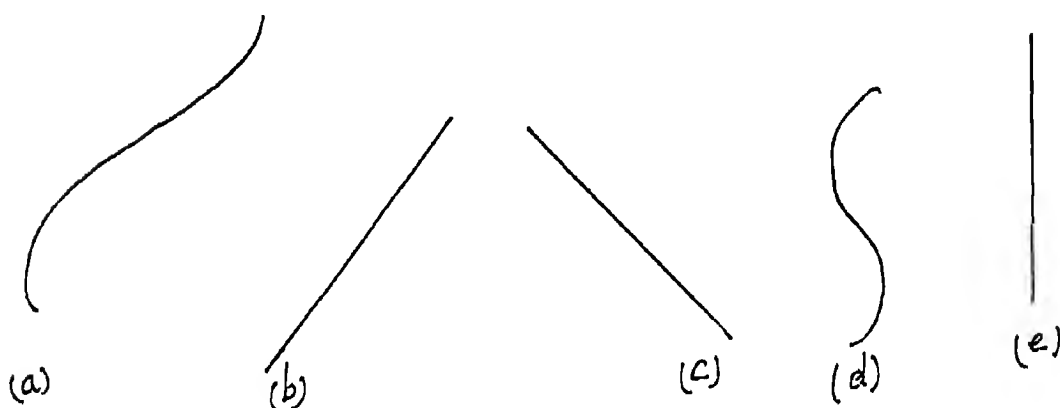
**Explanation:** The teacher is expected to give practice to the children for drawing straight and curved lines by moving a pencil along the edges of solids having flat and curved surfaces.

#### **Activities:**

- (i) Place a coin and trace its edge by moving a pencil along the edge. You get a curved line.
- (ii) Keep a book and draw a line along its edge you get a straight line.
- (iii) Take a thread and stretch it between your hands. Later let it be held loose. Here the teacher is expected to show a straight line and curved line with the above activity
- (iv) The above activity could be further extended by dipping the thread in ink and tracing a straight line and curved line.

**Further Activities:-**

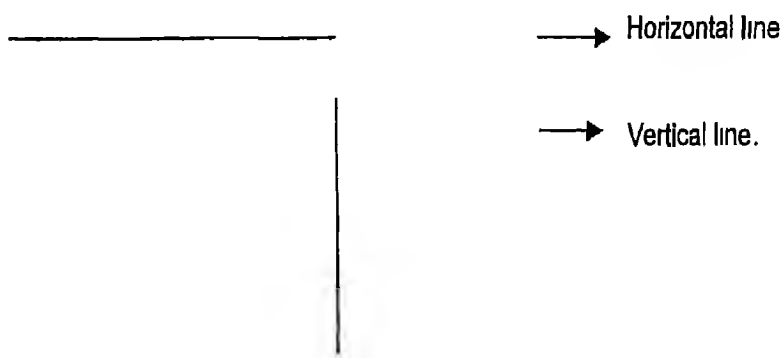
- (i) Ask the children to move the pencils along the bottom edges of an oil tin, a carrom-coin, a rupee coin, a match box, a pencil box etc , by placing them on a sheet of paper
- (ii) Give a collection of straight and curved lines and ask the child to pick out the straight lines from them.



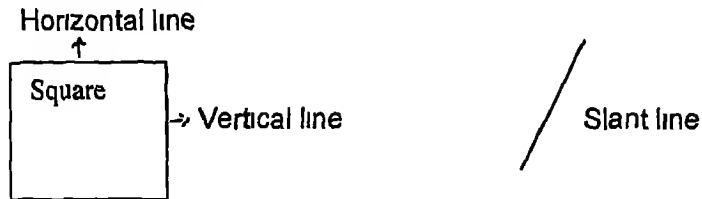
- (iii) Draw a few curved and straight lines
- (iv) Give a few solids like cube, cuboid, cylinder, cone and ask them to identify which can be used to draw
  - only straight lines
  - only curved lines
  - Both straight and curved lines

**3.1-4 Draws horizontal, vertical and slant lines**

Explanation. Teacher is expected to give the idea of vertical and horizontal lines



**Activities:-** Ask the children to keep a cube over the paper and move the pencil along edges of the bases.



Later ask them to identify the horizontal lines and Vertical lines in them. Then draw a line which is neither horizontal nor vertical and ask the children to compare them with horizontal and vertical lines. This straight line which is neither horizontal nor vertical is called a slant line.

**Further Activities:**

- (i) Make chits containing names horizontal, vertical and slant lines as shown in the figure. Put them inside a box.

HORIZONTAL LINE

VERTICAL LINE

SLANT LINE

Prepare different kinds of boards and write horizontal, vertical and slant lines over it.

Ask a child to pick up a chit. Call one child at a time and ask them to place it accordingly over the black board.

HORIZONTAL LINE

V  
E  
R  
T  
I  
C  
A  
L  
  
L  
I  
N  
E

S  
L  
A  
N  
T



### **Further Activities 2:**

- (i) Drawing a long dotted line.
- (ii) Tracing activities involving horizontal, vertical and slant lines.
- (iii) Giving incomplete pictures built with horizontal, vertical and slant lines asking them to complete it.
- (iv) Give the children figures having all kinds of lines and ask them to count the number of horizontal lines, vertical lines, and slant lines.

### **3.1-5 Draws (Free hand) shapes of square, rectangle, circle and triangle**

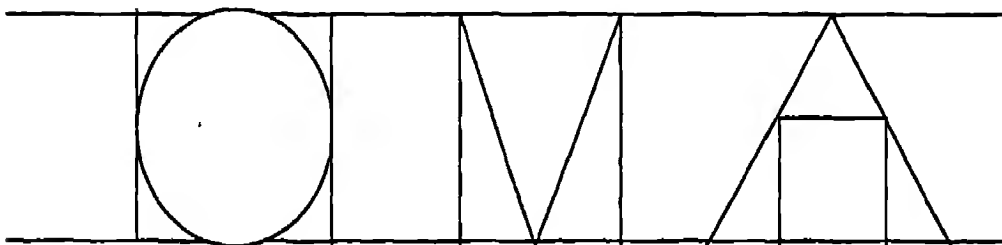
Explanation. With the help of various solids like cube, cuboid, cylinders, cones etc , the flat surfaces of them can be used to draw 2-dimensional shapes on a piece of paper which are called basic shapes.

Activities:(i) Drawing basic shapes using cutouts and objects available in the immediate environment.

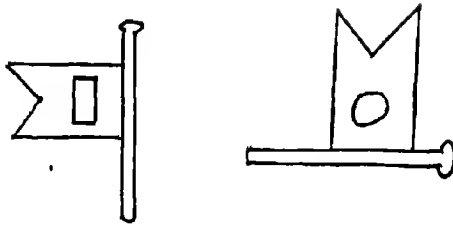
(ii)Tracing activities viz tracing out circles, triangles, squares etc along, with dotted lines and then continuing to draw free hand diagrams.

(iii)extending the series involving the four basic plane figures

Eg,-



(v) Copying of the given figure (See Fig 1)



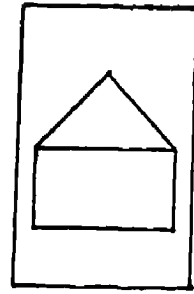
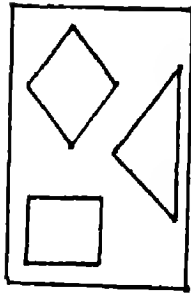
Recreation activities (aligning game)

- (i) Prepare 3 boards each embodying 5 holes cut of the same geometric shape arranged in ascending size and the insets with card-board painted in different colours as shown in Fig 2. Encourage the child to discriminate the size and geometric shapes and hence arrange them accordingly.



- (ii) Prepare two sets of cards. One set of cards displaying elementary geometrical shapes placed without any indication of order. The other set of cards having the same shapes structured to form a

composite shape Encourage the child to scan, analyse and pair the cards



**3.1-6: Draws shapes of triangles, squares rectangles, circles by paper folding and paper cutting.**

**Explanation:**

In order to ensure that the basic shapes are fixed in the child's mind, it is necessary that the teacher gives them practice in making these shapes with match-sticks, bangles, a paper folding, paper cutting etc. Enough opportunity may be provided to children to learn through experimentation and exploration

**Activity:**

(i) Take a sheet of paper. Fold its shorter side on to the longer side, cut off the excess, unfold. What you get is a square. See Fig 1.

Fold the square by bringing one of its corners on to the opposite corner, press, cut along the crease and obtain two pieces. Each piece is a triangle.

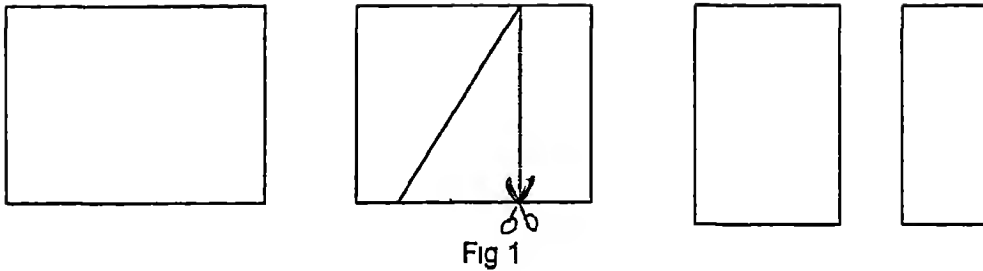
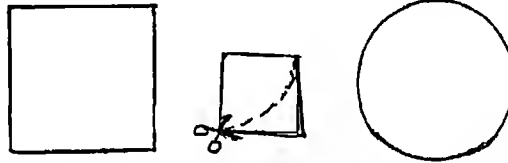


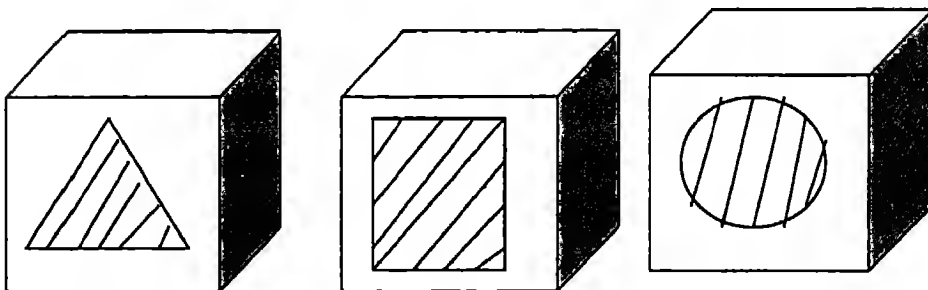
Fig 1

- (ii) Ask the children to bring matchsticks or straws from home, with the help of these show them how they can make squares, rectangles and triangles. Also ask them whether they can make a circle with the help of these sticks.

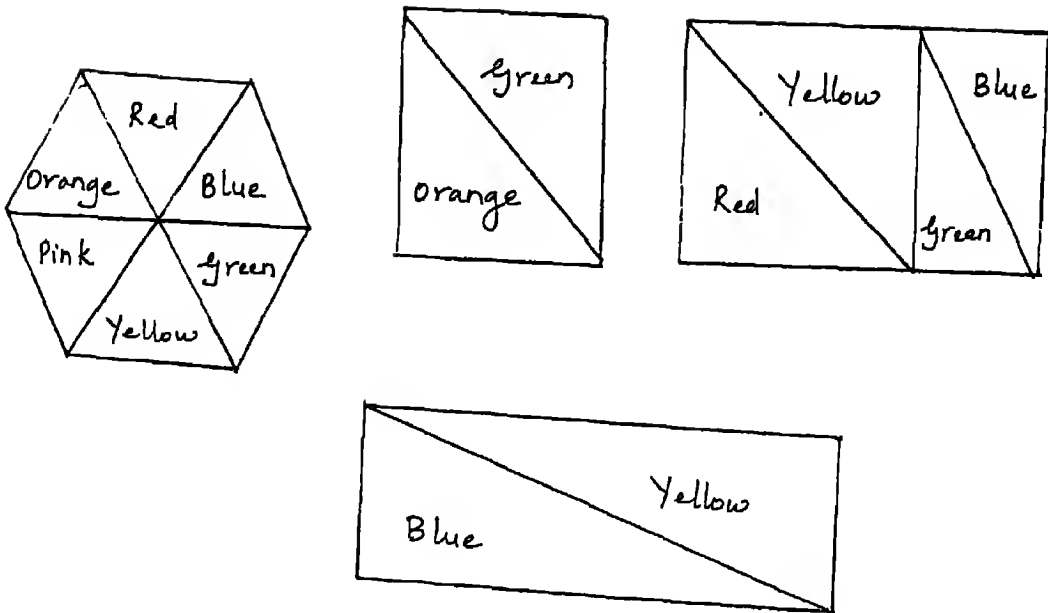


### Recreational Activities:

- (i) Ask the children to make rubber-stamps of various plane figures with the help of thermocole. When the stamps are ready, apply ink or paint over them and trace beautiful plane figures on the paper.



- (ii) The teacher is expected to make a number of shapes on colour papers and cut them out. Give each child a chart and ask them to use the given colour sheets to make various interesting patterns.



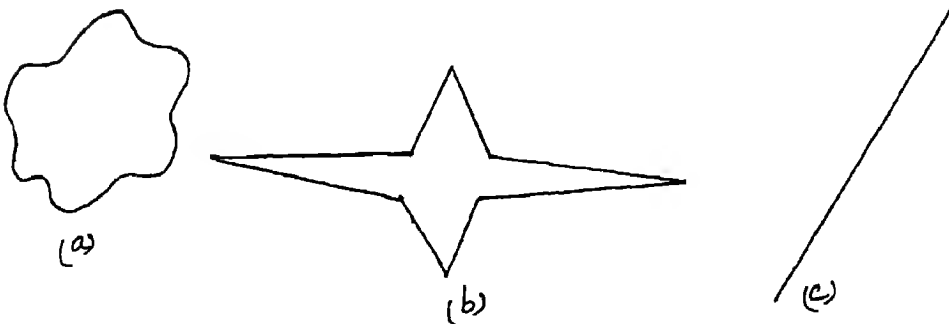
### 3.1-7: Recognizes closed and open figures

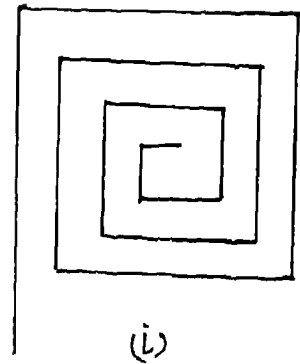
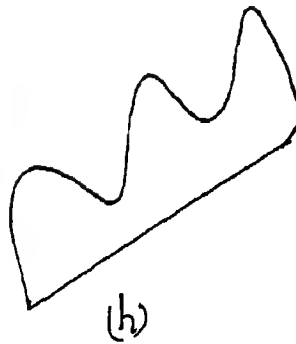
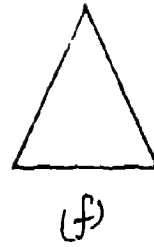
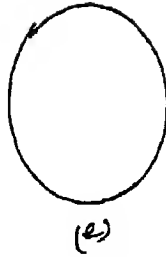
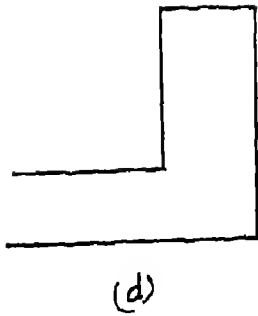
**Explanation** The figures traced out with the help of sharp edge of a pencil without lifting the pencil are called curves

Curves which end at the starting point are called closed figures, while the curves which do not end at the starting point are called open figures

#### Activities:

- (i) Draw a few figures on the board and call one child at a time to the board and ask them to trace over it and find out whether the given figure is closed or open





- (ii) with above figures ask the children to classify them as closed figures and open figures

Closed	Figures	Open	Figures
--------	---------	------	---------

- (iii) Ask the children to draw a few closed and open figures on their own

- (iv) Ask the children to collect a few articles from home which can be used to draw open figures

Eg A broken gasket, broken bangle an open watch strap, a broken ring etc ,

### 3.1-8: Makes simple patterns and models out of given shapes:

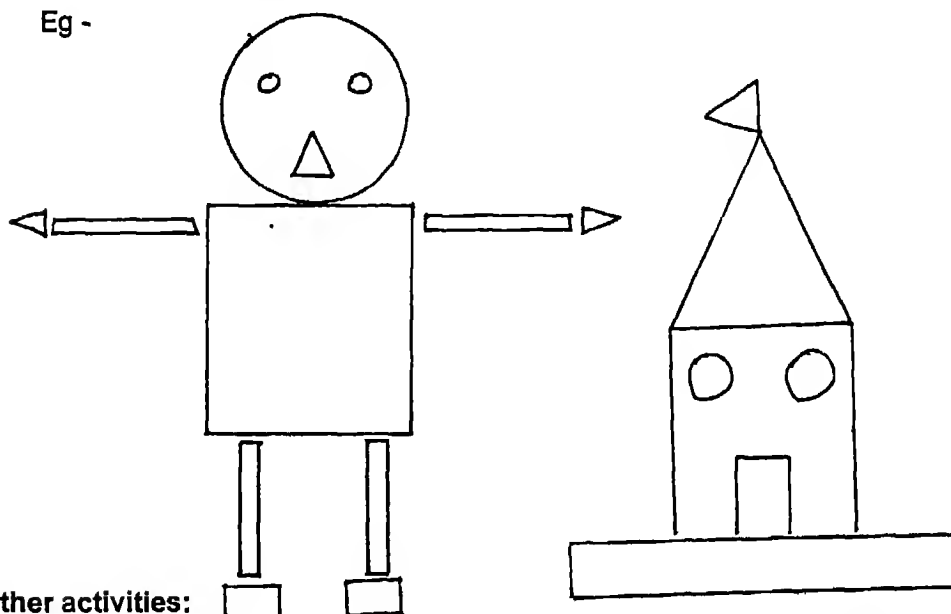
The 2 dimensional basic shapes have been a boon in the creation of thousands of designs/patterns on our clothes, bed sheets, sarees, table-covers etc , Children may be provided opportunities to observe such designs and patterns on clothes, sarees, table covers, buildings etc , The teacher is

expected to encourage children to observe, copy and draw a lot of patterns and models

**Activities:**

- (i) Teacher is expected to draw a few patterns on the board with the help of the four basic shapes. In the same manner encourage the children to create more such patterns.

Eg -



**Further activities:**

- (i) Take the children to the school ground and ask them to identify any of the four basic shapes if present over there.
- (ii) Ask the children to cut-out the four basic shapes on a card board and with the help of these make their own models.
- (iii) Take the children to a nearby historical monuments (Temple, church, monuments and ask them to identify the basic shapes in the paintings over the walls

**Recreational Activity:- (Aligning game )**

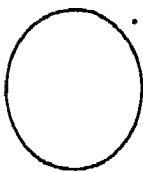
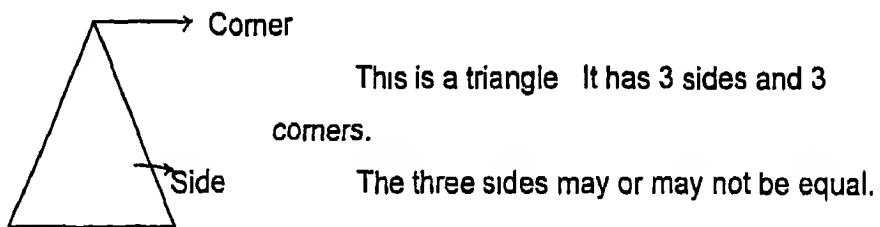
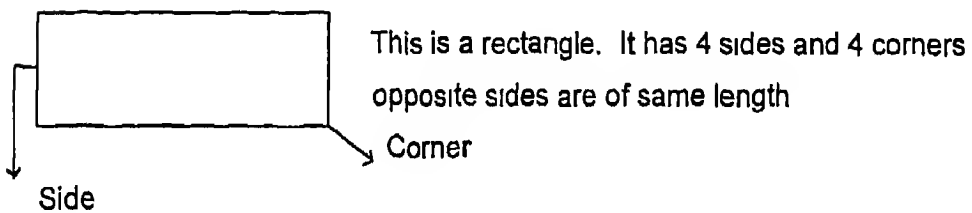
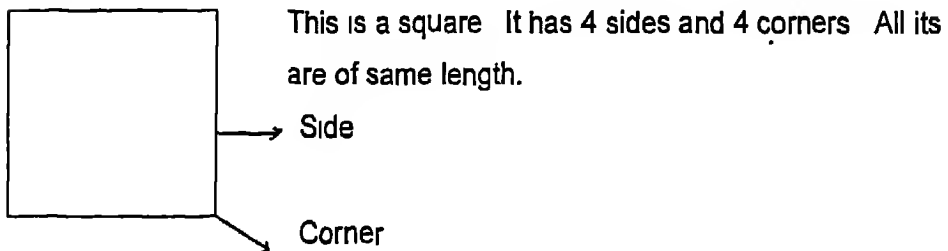
The teacher can plan a game involving 2-4 students on the four basic shapes as follows:

The teacher is expected to prepare the recess-boards and the corresponding geometric shapes of different colours/shapes Administer it to

the children and encourage them to align the shapes over recess boards. The one who completes it first and correctly will be declared the winner.

### 3.1.9. Discovers and narrates simple characteristics of shapes:

Explanation:-



This is a circle. It has no corner.

Following questions may be used for evaluation.

- (i) How many sides and corners does a square have?
- (ii) How many corners does a triangle have?
- (iii) How is a triangle different from a square?
- (iv) Can you make a square by using triangles?
- (v) How many corners does a circle have?



**Further Activities: (making cut-outs)**

- (i) Teacher has to prepare a few cut-outs of rectangles, squares, triangles and circles and give it to the children to draw the figures in their note book
- (ii) Teacher is expected to draw diagrams involving only the 4 basic shapes on flash-cards and ask the children to count the number of squares, rectangles, triangles and circles
- (iii) The above activity could be extended further into a colouring activity in which the teacher assigns a particular colour for a figure and the children are expected to colour the figures accordingly
- (iv) Give the children a few solids and ask them what would be the plane figure which could be obtained by moving a pencil along its edges. The children can be asked to verify their answer by tracing out the figures on the paper using the solids given to them.  
eg - A post card, a pencil box, a match box, a dice, a set square, a bangle, a carrom pawn, a rupee coin etc.,

**Recreational Activity:- (Matching game)**

The teacher can frame a game involving 5 students as follows:

Make a die with each face embodied with a plane figure namely square, rectangle, triangle, circle, quadrilateral, pentagon. Make 30 cards subdivided into 5 groups, each group reproducing the same set of shapes, embossed in the die. The cards must be made in different colours and the presentation of shapes should vary between solids and outlines. Each child should be given a turn to throw the die in sequence and should present the card matching to that on the face of the rolled die from his/ her collection of cards

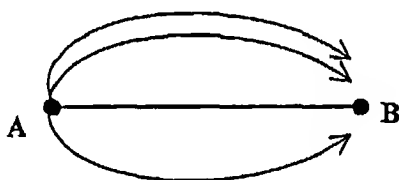
### Sub Unit 3.2 CLASSIFICATION OF GEOMETRICAL FIGURES AND SYMMETRY

- 3.2.1 Straight lines and curved lines.
- 3.2.2 Classification of the figures (Triangle, quadrilateral and circle).
- 3.2.3 Symmetrical and non-symmetrical shapes (Paper foldings).
- 3.2.4 The line of symmetry and the designs (paper foldings).
- 3.2.5 Terms related to circles (center, radius, diameter, chord and circumference).

#### 3.2.1 Straight lines and curved lines

Here we give some activities involving straight and curved lines.

**Activity 1 .** The teacher can take the students to the playground and fix two points A and B as shown in the figure. Make the students walk from A to B by different paths. Who is walking on the shortest path ?

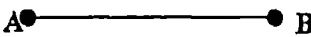


What is the shortest path from A to B ? It will be clear to the students that the shortest path from A to B is the straight line (path) and this is the line segment AB. All the other paths are the curved paths

Ask them to draw the paths on a paper sheet and classify them as straight and curved paths

a) Straight path

Line segment AB



b) Curved path

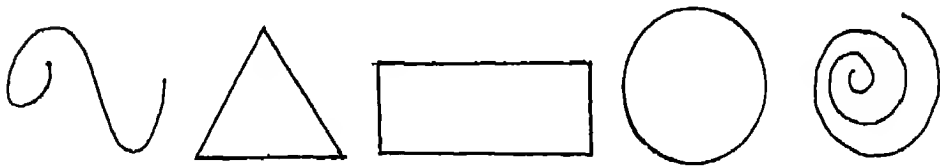
Curved line AB



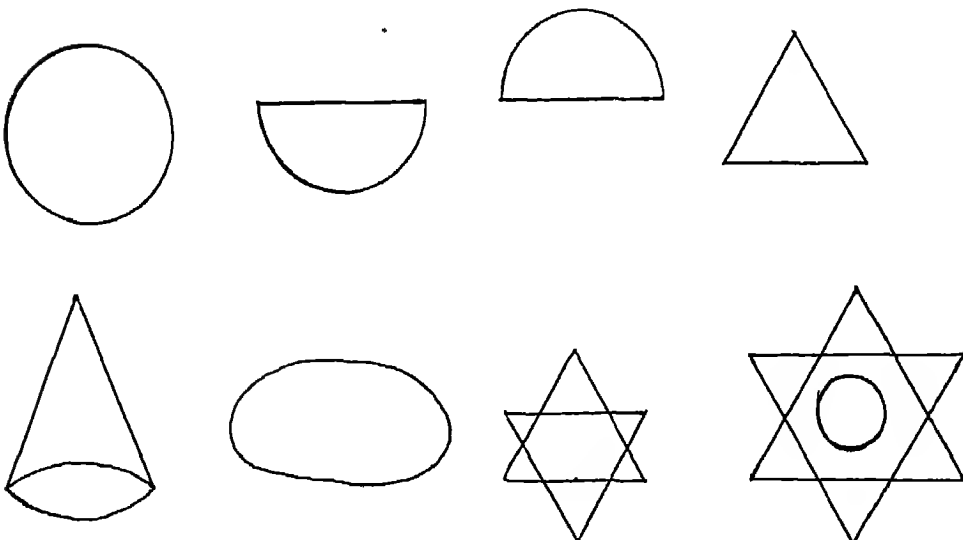
#### Questions

1. How many straight paths (line segments) are there from A to B ? Draw them

- 2 How many curved paths (curved lines) AB can be drawn? Draw them
- 3 Draw the straight line (path) from a place to another place on the sketch map of India
4. Draw the other curved paths also between these two places.
- 5 Identify the figures containing straight lines and curved lines in the following



- 6 Draw any two figures consisting of only straight lines.
7. Draw any two figures consisting of only curved lines.
- 8 Draw any two figures consisting of both curved lines and straight lines
9. In the following figures show curved lines in blue colour and the straight lines in red colour








2. Classification of figures

Activity 1:







Students can be asked to prepare triangles, squares, rectangles, circles, half circles and quarter circles by paper folding and paper cutting. If this activity is already done in the previous class, then they can prepare charts, here again and give the names for each figure as shown below (after tracing them on the paper).

Chart I

				
Rectangle	Square	Circle	Half circle	Quarter circle

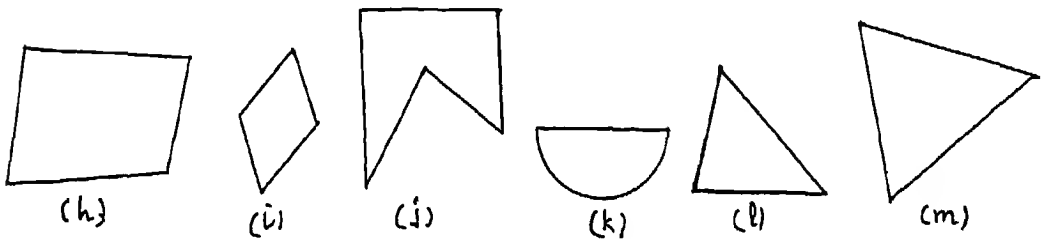
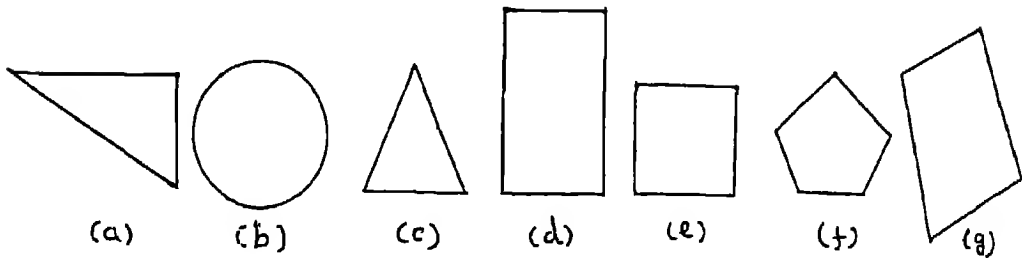
Another set of figures containing only the line segment (straight lines) can be drawn on the paper and the corresponding names can be given.

Chart II

					
Rectangle	Square	Triangle	Triangle	Triangle	Quadrilateral

- Q.1 What is the difference between the figures in Chart I and Chart II ?
- 2 How many line segments are needed to draw a triangle ?
- 3 How many line segments are needed to draw a rectangle (or a square)?

4. How many line segments are needed to draw a quadrilateral ?
5. Look at the following figures and identify (name) the triangles, quadrilaterals and circles.



### 3. Symmetry

Tiger! Tiger! Burning bright  
 In the forests of the night,  
 What immortal hand or eye  
 Dare frame thy fearful symmetry ?

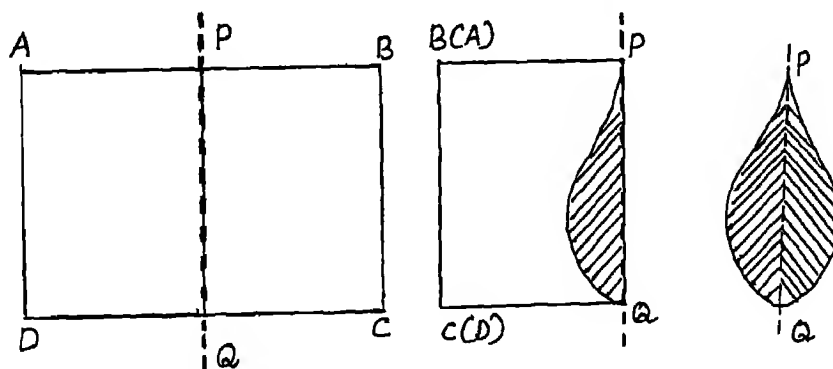
( William Blake, "The Tiger" from Songs of Experience)



This was one stanza of the song written by the famous English poet William Blake enjoying the symmetry in the face of a Tiger (Even though it is quite fearful).

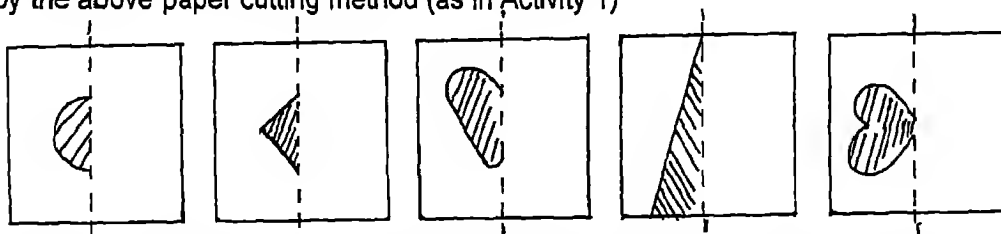
Also, there are many more things in nature, which are symmetrical, the teacher can show those symmetrical objects to the students.

**Activity 1:** The teacher can ask the students to fold a rectangular sheet of paper in the middle, vertically at the dotted lines as shown here in (i)



And cut along the outlines of the shape as shown in fig (ii) and then unfold the paper. They will get a cut out in the form of a leaf. This figure is symmetric about the line PQ (because if you fold it along PQ, the one half of it will overlap the other half). This leaf is *symmetrical* about the line PQ and the line PQ is called the *line of symmetry*.

**Activity 2:** Ask the students to prepare several different symmetrical figures by the above paper cutting method (as in Activity 1)

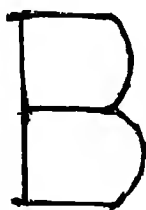


**Activity 3** Ask the students to write the letter capital A on the graph paper and find out whether it is symmetric. If so, what is its line of symmetry?



If they fold it at the dotted line as shown here, will the right side and left side overlap?

**Activity 4** · Ask the students to write the letter B on a graph paper and find out whether it is symmetrical. If so, what is its line of symmetry ?

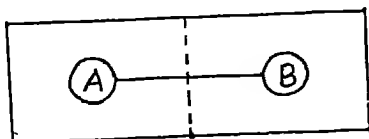


**Activity 5** · Ask the students to write letter P on a graph paper and find out that P is not symmetrical. It has no line of symmetry



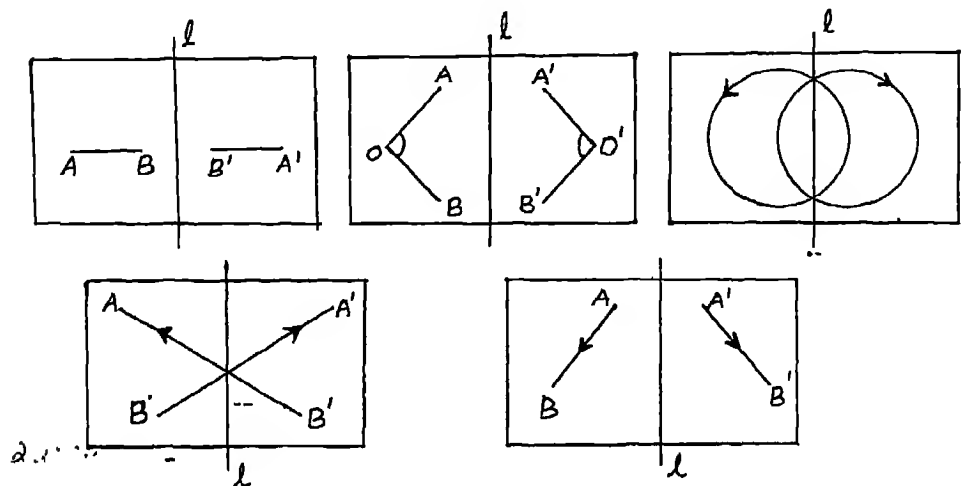
P is non symmetrical in shape. Can we find out the other letters which are symmetrical and non symmetrical ?

4. *The line of symmetry* and designs If we fold the paper along the line then A will fall on B



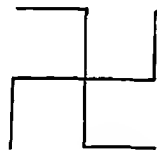
This line / is called the line of symmetry of the given figure

Now, we give some figures which are symmetrical about a line  $l$ . The teacher can ask the students to prepare similar figures by folding the paper along the line  $l$  and tracing it on the other side.



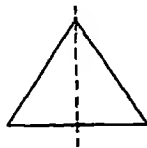
Question 1 :

Find out whether the swastika given below is symmetric ?

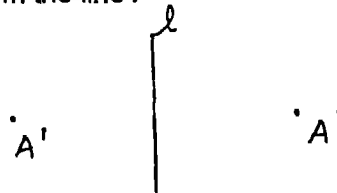


If so find the line of symmetry (by folding the paper)

2. Find out whether the triangle ABC (whose all three sides are equal) is symmetric? Find its lines of symmetry? How many lines of symmetry are there ?



- 3 If two points A and A' are symmetric about a line  $l$  then what can you say about their distances from the line  $l$  ?





4. Fold a square shaped paper vertically first, then fold it again in the middle horizontally. Draw any picture on it and cut it along the outline of the picture, unfold the paper and find whether it is symmetrical ? How many lines of symmetry do you get ?

5 Draw the figure of a person standing straight. Is it symmetrical? Where is its line of symmetry ?

6 Whether our two palms are symmetrical?

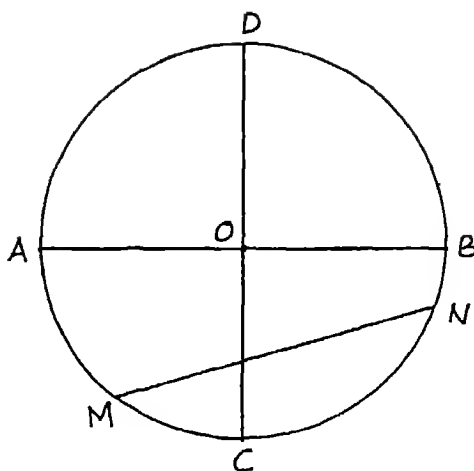
### 3.1.5. Circle

In the previous classes, the students have learnt to draw the circles using objects like a *bangle*, a rupee coin, a carrom coin, etc

The above activities may be repeated here again to reinforce the concept.

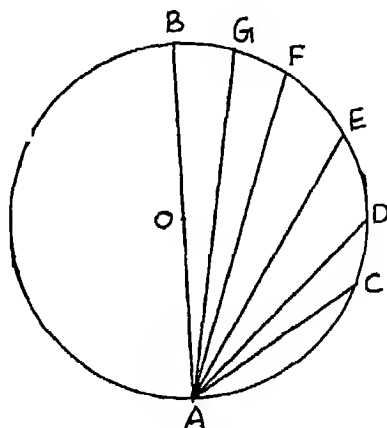
1. Ask the students to keep any round object (bangle, coin, etc.) on a piece of paper and move the pencil along the boundary of the object to get a circle. Ask the students to name it as "circle". Cut the piece of paper along the boundary (circle) Fold this paper (circle) twice so that we get two intersecting straight lines AB and CD Call the intersecting point as 'O' and O is the center of the circle. OA is called the radius. AB is called the diameter. If M and N are any two points on the circle, then MN is called a *chord* of the circle.

Centre of the circle	.	O
Radius of the circle	:	OB
Diameter of the circle	:	AB
Chord of the circle	.	MN



The following additional clarifications may be given to the students.

1. CD is also a diameter of the circle because CD also passes through the center O of the circle.
2. MN is not a diameter because MN does not pass through the center O.
3. OA, OB, OC and OD are all equal. (Do an activity of measuring them with a thread and let the students verify it) and it is called the radius.
4. If M and N are any two points on the circle, then the line segment MN is a *chord* of the circle. The diameter is also a chord of the circle. The diameter has to pass through the center of the circle. The other chords need not pass through the center.
5. Longest chord of the circle is the diameter. In the figure, AC, AD, AE, AF, AG, AB are all chords of the circle. But AB (diameter) is the longest chord.



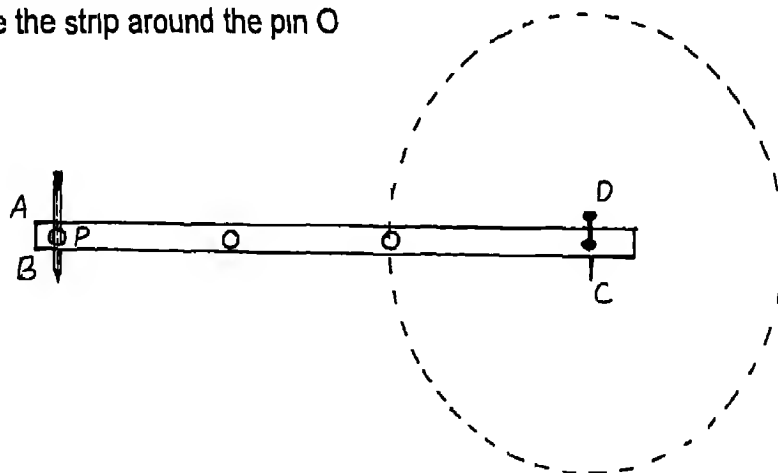
The following activities may be done to reinforce the concepts

### Activity 1

Fix a peg in the ground at O. Tie one end of a piece of twine thread to it to the peg and hold the other end P with a marker of the thread tight touching the ground. Move this end about the peg one complete round. The curve traced on the ground will be a *circle*.

- 1 What is the radius of this circle? (Ans. The length of the thread = OP).
- 2 What is the diameter of this circle? (Ans. Double the radius).
- 3 What happens to the circle as you increase the length of the thread?  
(The circle becomes bigger)

**Activity 2** ABCD is a rectangular cardboard strip. Fix one end of the strip by means of a pin on a sheet of paper at O. Fix a pencil to one of the holes P. Move the strip around the pin O.



- 1 What is the name of the curve traced? (Circle)
2. What is the radius of this circle? (OP).
- 3 As you change the pencil from one hole to another what happens to the curves? (Ans: The size of the circle becomes bigger as the radius increases.)

**Activity 3** · Now, show the children, how to draw the circles using a compass.

**3.3: LINE, PLANE AND ANGLES**

- 3.3-1 :** Illustrates concept of a plane using appropriate examples from her/his immediate environment.
- 3.3-2 :** Makes distinction between a ray and a line segment.
- 3.3-3 :** Understands that an angle is made of two rays/line segments.
- 3.3-4 :** Identifies angles occurring in various shapes.

Sub Unit: 3.3.1

Illustrates concept of a "Plane"

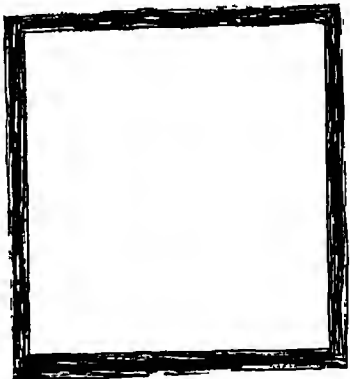
Using appropriate examples from her/ his immediate environment.

**INTRODUCTION:** The teacher can take the following objects in eliciting that " Some objects are having flat faces and some objects are having curved faces". Further he can state that "the flat face is called a plane"

Example:

1) Pencil Box	Cuboid	Flat face
2) Chalk Box	Cuboid	Flat face
3) Ball	Sphere	Curved face

Look at the picture of a slate shown in the figure. The slate has a flat surface. This flat surface is a plane Plane surface extends in all directions



EXPLANATION:

The teacher is expected to draw the attention of the students about Geometric Solids, some having flat surfaces and some having curved surfaces, because they have already learnt in previous classes.

Examples:

Fig 1. CUBOID

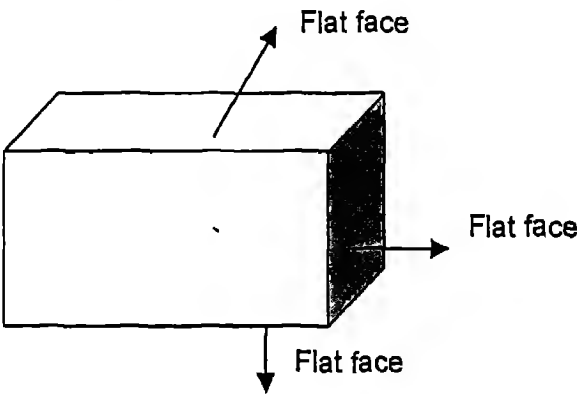


Fig 2: SPHERE

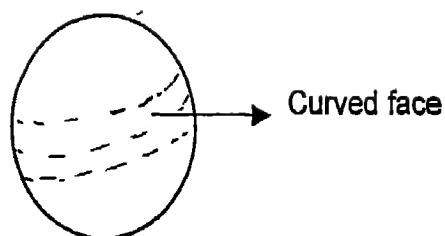
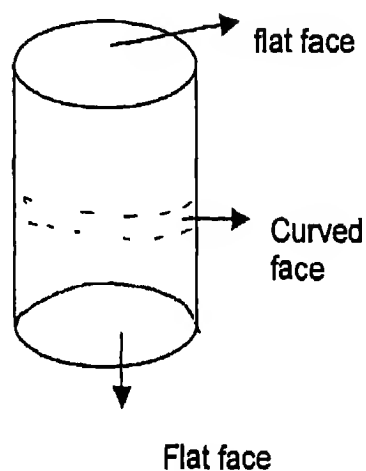


Fig 3 · CYLINDER·



With these examples the teacher can state, "the flat faces are called Planes".

Further he has to mention some more examples for planes like

- a) Surface of the Black Board
- b) Top surface of the Table.
- c) Surface of the Classroom wall.

**Activities' suggested to do in the class room with the help of the students.**

I Collect the following objects and elicit the number of flat surfaces (planes) and curved surfaces that they are having.

	Name of the Object	No. of the flat faces	No. of the curved faces
1.	Instrument Box	6 – flat face	
2.	Cuboid	—, —, —	
3.	Match Box	—, —, —	
4.	Brick	—, —, —	
5.	Cube	—, —, —	
6.	Dice	—, —, —	
5.	Sphere		1- curved face
6.	Ball		—, —, —
7.	Cricket Ball		—, —, —
8	Metallic bob		—, —, —
9	Volley Ball		1- curved face
10	Badminton Ball		—, —, —
11.	Tennicoit ball		—, —, —
12.	Basket Ball		—, —, —
13.	Rubber Ball		—, —, —
14.	Tennis Ball		—, —, —
15.	Bucket	1-flat face	1-curved face
16.	Pencil	2-flat face	1-curved face
17.	Ruler	—, —, —	—, —, —
18	Rupee coin	—, —, —	—, —, —
19	Triangular Prism	5-flat face	
20.	Metre Scale	6-flat face	
21.	Cone	1-flat face	1-curved face
22.	Top surface of the water containing beaker	1-flat face	
23.	Striker (Carom Board)	2-flat face	1-curved face
24	Metallic rod	—, —, —	—, —, —

II Classify the following objects into objects having flat face, curved face and both.

- a) Suit case      b) torch light      c) beads      d) Refil of the pen



- e) Gas cylinder      f) foot ball      g) Half a metre scale      h) 50 p-coin  
 i) Battery cell      j) Rubber      l) Note Book      m) Maths Text Book.

**III Labelling the following as objects having flat face, curved face and both**

- a) Circular dial      b) Cncket ball      c) Cuboid      d) Cube      e) Cone  
 f) Sphere      g) Cylinder      h) Triangular Prism

**Further activities outside the classroom:**

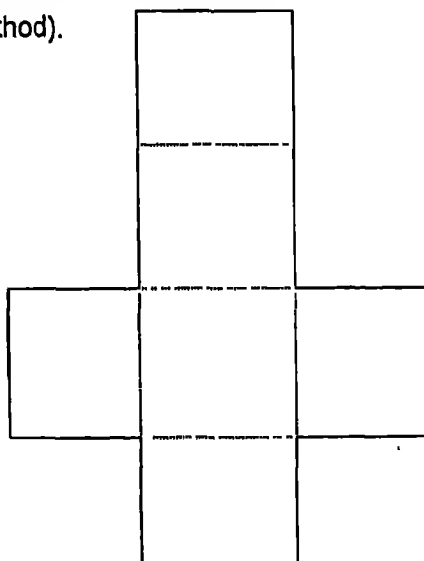
The teacher can take the students to the following places and ask them to list out the objects that they come across. And classify the objects into solids having flat face, curved face and both

- 1) Visit to a stationary shop
- 2) Visit to a medical store.
- 3) Visit to a provision store.
- 4) Visit to a Brick Factory
- 5) Visit to a Book Stall.
- 6) Visit to Lab.
- 7) Visit to a post office.
- 8) Visit to a sports field

**Enrichment activities:**

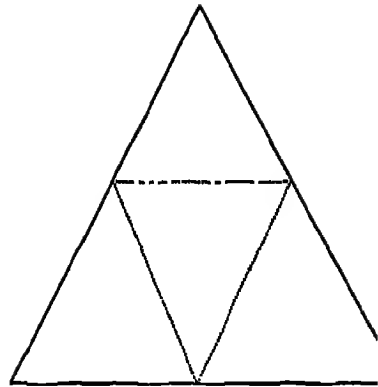
It is suggested that the teacher may help the students to draw the following figures on the sheet of a paper and prepare the following models, count the flat faces (planes) and curved faces and both. Verify the answer given. (For making models use folding & paper cutting method).

- a) Draw six squares equal in area on a sheet of paper as shown in the fig. Cut the paper in the boundary. Fold the paper on the lines dotted in the figure to get a cuboid, Now count the number of flat faces, (planes) and curved faces.



[Ans: 6- flat faces and 0-curved faces]

- b) Draw four equilateral triangles equal in area on a sheet of paper as shown in the fig. Cut the paper in the boundary. Fold the paper on the dotted lines marked in the fig (b) to get a triangular prism. No count the No. of flat faces (planes) and curved faces



[Ans 4- flat faces and 0-curved faces]

- c) Take a rectangular sheet of paper and roll it to get a hollow cylinder shape. Paste the ends (to get a hollow cylinder ). So that it will remain as a hollow cylinder. Count the curved surfaces. Is it having flat surfaces? Now we shall close the top and the bottom by means of circular dials of paper Count the flat surfaces also.  
(2-flat surfaces, 1- curved surface)

- d) Take a rectangular sheet of paper and fold it to get a (3 rectangles equal in area) hollow triangular prism Paste the ends so that it will remain as a hollow Triangular prism. Count the flat faces Now we shall close the top and bottom by means of triangular dials of paper Finally count the flat faces Will it be more in number or less in number if we compare, it with the first answer?

(Ans 5- flat faces)

**SUB UNIT:3.3:2: Makes distinction between a ray and a line segment.**

**Introduction:**

It is suggested that the teacher asks some questions about "Line Segment" to recall the previous knowledge. Draw the attention of the students through the following activities.

- a) Draw any line segment and mark it as AB.

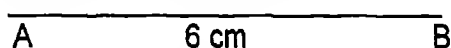


Fig (i)

What are the end points?

A & B

- b) Extend the line segment AB on both directions.



Fig (ii)

Compare fig (i) and fig(ii)

Can we call fig (ii) as a "Line Segment"? No, this is a line segment extended in both the directions endlessly. We call this as "A Line".

The arrow marks indicate that the line segment is extended in both the indicated directions

- c) Mark a point "O" in between A & B as shown in the fig(iii)

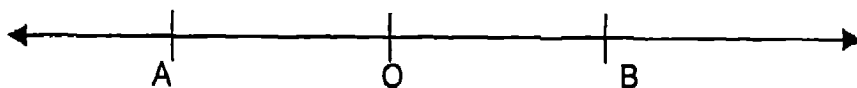
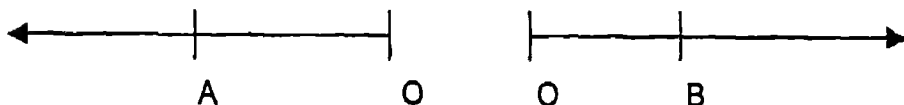
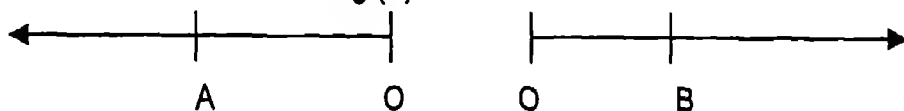


Fig (iii)



How many parts do you get?

Two parts ie OA and OB.

Compare the part OA & OB with fig(ii).

Will it be the same? - No.

Can we call the part OA & OB as a line?

No, this is a part of a line.

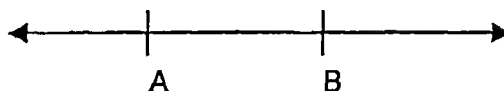
We can call this part OA or OB as "Ray".

### 1) A Line Segment:



It is a part of a line. It has a definite length. It has two end points. This can be represent as  $\overline{AB}$ . This can be drawn on a sheet of paper.

### 2) A Line:



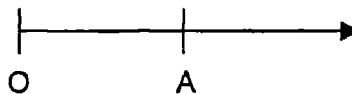
It has no end points.

It cannot be drawn on a sheet of paper

But one can represent it by a diagram

It does not have a finite length. It can be represented as  $\overleftrightarrow{AB}$

### 3) A Ray:

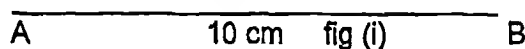


A portion of line which extends in only one direction from a point, is a Ray. The Point "O" is called the end point of the ray. A ray has only one end point. It can be represents as  $\overrightarrow{OA}$

### Explanation through an activity:

It is suggested that the teacher should draw the attention of the students about the following activity.

- 1) Draw a line segment of length 10 cm on a sheet of paper with the help of a scale and pencil. Mark it as AB.

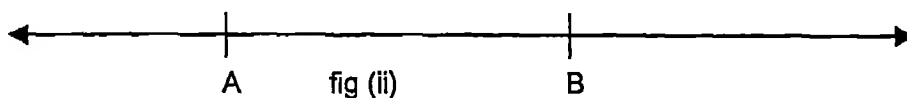


Which are the end points? - A & B

How many end points are there? - Two

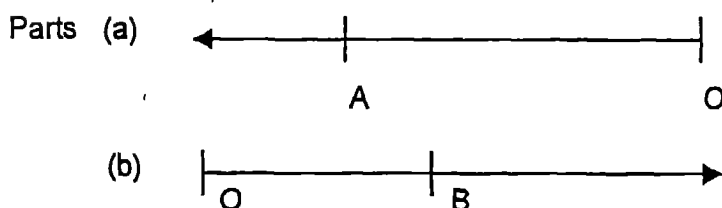
A line segment AB can be represented as  $\overline{AB}$  (or  $\overline{BA}$ ).

- 2) Extend the line segment AB on both sides in both directions as shown in the fig.



Compare fig (i) & (ii), We can easily see that It has no definite length This is a line A line cannot be drawn on a sheet of paper, but this can be represented on a sheet of paper. A line can be represented as " $\overleftrightarrow{AB}$ " we can denote symbolically as  $\overleftrightarrow{AB}$  or  $\overleftrightarrow{BA}$ .

- 3) Mark a point "O" in between A & B as shown in the fig (iii)



The point "O" divides  $\overleftrightarrow{AB}$  into two parts.

Name these two parts? -  $\overrightarrow{OA}$  &  $\overrightarrow{OB}$

Name the end point of part - "O"

Similarly name the end point of part -OB? -"O".

The part OA or OB can be called as "A Ray"

A ray has only one end point. It can be extended in one directions through one point only (other than the end point). A ray OA or OB is represented as  $\overrightarrow{OA}$  and not AO.

This activity helps the students to get a clear idea about a line segment, a line and a ray.

Further the teacher can utilize the results of the above activity in writing the distinction between a line segment, a line and a ray.

#### Distinction:

A Line Segment		A Line		A Ray	
1.	We can draw a line segment on a paper.	1	We cannot draw a line on a paper but it can be represent by a diagram	1	We cannot draw a ray but can represent it by a diagram.
2.	It has a definite length.	2.	It does not have a definite length.	2.	Does not have a definite length.
3	It has two end points.	3.	It has no end points.	3	It has only one end point.
4.	$\overline{AB}$ represents a line segment.	4.	$\overline{AB}$ represents a line.	4	$\overrightarrow{OA}$ represents a ray
5.	We can named as $\overline{AB}$ or $\overline{BA}$	5.	We can named as $\overline{AB}$ or $\overline{BA}$	5.	We can named as $\overrightarrow{OA}$ only

**Activities suggested to do in the class room with the help of the students:**

- 1) Draw as many line segments ( of various lengths) as you can on a sheet of paper & ask the students to measure.
- 2) Ask the students to draw line segments by using set squares, protractor base, straight edges of a text book, chalk box, instrument box, pencil box, cuboid, cube, triangular prism.
- 3) In the above examples ask the students to extend line segments on both sides to get lines.

- 4) Ask the students to mark a point on the lines they got already. This gives rays.

#### Activities outside the classroom:

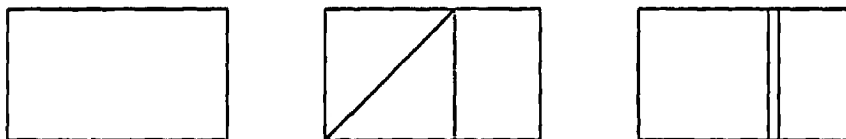
The teacher can take the students to the following places & ask them to list out the objects that they come across, so that they are in a position to draw line segments by using the above objects in turn lines and rays.

- 1) Visit to school play ground.
- 2) Visit to a stadium.

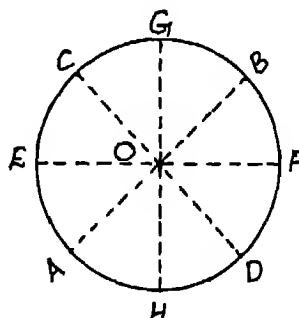
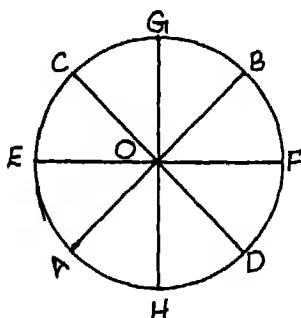
#### Further activities:

The teacher can ask the students to do the following activities in their homes

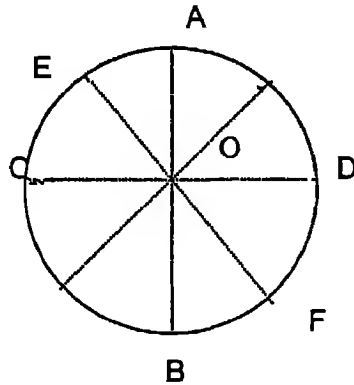
- 1) Take a sheet of paper. Fold its shorter side on the longer side. Cut off the excess, unfold. What you get is a square. Measure the sides of the square. (All sides are having same length)



- 2) Draw a circle of radius 5 cm. Mark the center as O. Draw lines AB, CD, EF, GH as shown in the figure. Measure the same. What do you infer? (Same length- i.e. Diameters of a circle are having same length). Further measure OA, OB, OC, OD, OE, OF, OG, OH. What do you infer? (Same length- i.e. Radii of a circle are having same length)



Draw a circle of radius 6 cm. Mark the center as O. Cut the paper on boundary. Fold the paper on dotted lines as shown in the fig. Observe the folding AB, CD, EF, GH. Measure the same. (All foldings are of the same length).



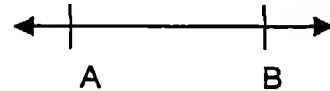
The teacher, can draw the attention of the students about the following statements

We can name the line segment as shown below.



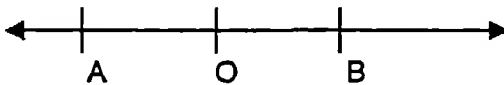
Line segment AB & line segment BA.

We can name the line as shown below



Line AB & line BA.

We get a line from two rays which have opposite directions and unknown endpoint ( $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ )



We can name the ray  $\overrightarrow{OA}$  only or  $\overrightarrow{OB}$  only.



A line segment has definite length



- 6) A line segment has two ends
- 7) A line segment can be drawn on a sheet of paper.
- 8) A line does not have a definite length
- 9) A line cannot be drawn on a sheet of paper, but a line can be represented by a diagram.
- 10) A ray does not have a definite length
- 11) We cannot draw a ray but can represent it by a diagram.
- 12) A ray has only one end point.
- 13) A line has no end points
- 14) We can mention the following units of length for example of a line segment.

(i)

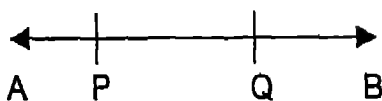
- |         |               |                |
|---------|---------------|----------------|
| a) 1 cm | d) 1 ft       | g) 1 mile      |
| b) 1 m  | e) 1 Yard.    | h) 1 inch      |
| c) 1 Km | f) 1 furlong. | i) 1 light yer |

- ii) The sides of a given square
- iii) The sides of a given equilateral triangle
- iv) The sides of a given Rhombus
- v) The opposite sides of a given rectangle

14. Mark two points P & Q on line A B

The distance between P & Q is nothing

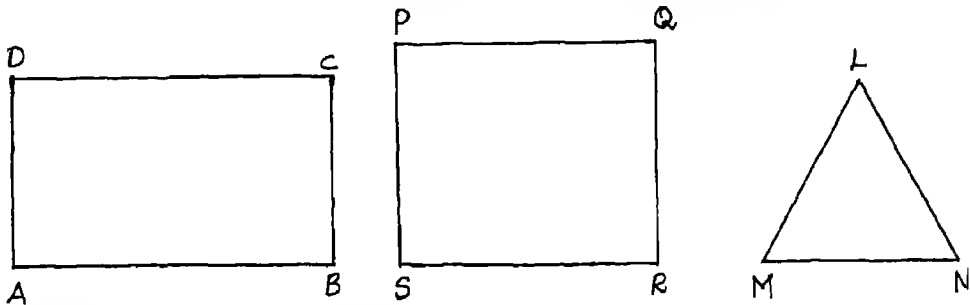
But a "line Segment"



**Sub-Unit 3.3.3 : Understands that an angle is made of two rays/line segments.**

**Introduction**

It is suggested that the teacher asks the students to draw the following figures on a sheet of paper and answer the questions given below



1. How many line segments are there in Fig.1 , Fig 2 and Fig 3? And name them  
 In fig 1 line segments : 4 – { AB, BC, CD, DA }  
 In fig 2 line segments : 4 – { SR, PQ, QP, PS }  
 In fig. 3 line segments : 3 – { MN, NL, LM }
  
2. How many corners are there in fig 1, fig 2 and fig 3? List out separately  
 In fig 1 corners – 4 i.e A, B, C and D  
 In fig 2 corners – 4 i.e S, R, Q and P  
 In fig 3 corners – 3 i.e M, N and L.
  
3. In fig 1
  - a) What are the end points of line segment AB ? [A & B]
  - b) What are the end points of line segment BC ? [B & C]
  - c) What is the common end point of line segments AB and BC ? [B]
  - d) Can we call the figure ABC as a corner ? Yes, ABC is a corner.

Here the two line segments AB and BC have B as a common end point. In other words, the line segment AB meets the line segment BC at B. We call ABC as an angle and denote it as  $\hat{A}BC$  or  $\angle ABC$ .

Is DAB an angle ? Yes.

Can we find the line segments with a common end point ? Yes.

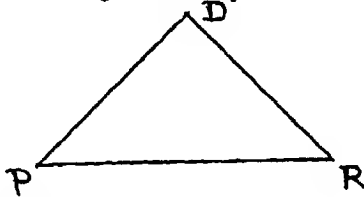
Similarly what are the other angles ? and the line segments which make the angle in Fig.1 ?

1. Two line segments with the common end point form an angle.
2. Two rays with the same end point form an angle.

### Explanation :

- I. The teacher can use the English alphabets in explaining the concept of an angle
- II. The teacher asks the students to move a pencil along the straight edges of a set square to get a triangle DPR as shown in the figure and answer the questions below.

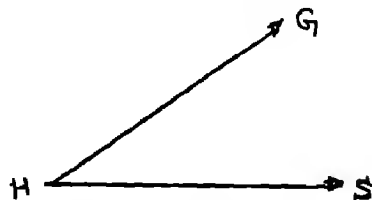
1. State the 3-line segments ? [DP, PR and RD].



2. What is the common end point of DP and PR ? [P]
3. Name the angle when DP meets PR at P ?  
[Angle DPR or  $\hat{D}PR$  or DPR.

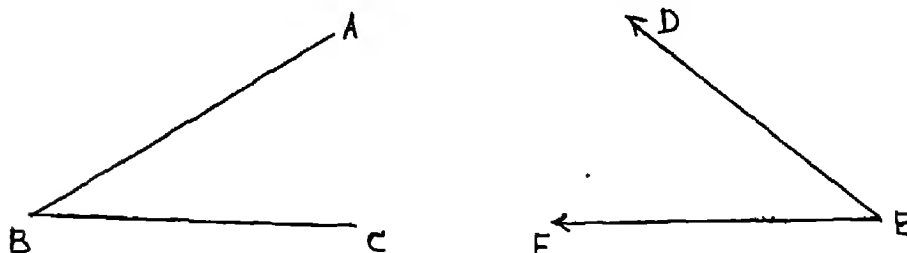
- III. The teacher asks the students to draw two rays on a sheet of paper, having the same end point to get an angle, as shown in the figure (by using a scale and pencil) and then he asks the students to answer the following questions

1. Name the two rays. [HG and HS].
2. Name the common end point? [H]
3. "H" is a common end point of two rays HG and HS. We call this as "vertex". H is the vertex of rays HG and HS.
4. Name the angle [  $\hat{GHS}$  or GHS or Angle GHS ].



V. The teacher should draw the attention of the students to the following:

a) Figures; b) Statements, c) symbols or Notations



1. ABC is an angle – the common end point B should be in between A and C.
2. DEF is an angle – the common end point E should be in between F and D
3. The symbol or notation of an angle is " $\angle$ " or " $\hat{\phantom{A}}$ ".
4. An angle ABC is written as  $A\hat{B}C$  or  $\angle ABC$  symbolically.
5. An angle DEF can be written symbolically  $\angle DEF$  or  $\angle FED$ .
6. To name an angle, we use capital letters of English alphabets
7. The  $A\hat{B}C$  or  $C\hat{B}A$  can also be written as  $\hat{B}$  or B
8. The  $\angle DEF$  or  $\angle FED$  can also be written as  $\hat{E}$  or E.
9. Common end-points of two line segments or rays is named as "vertex" of the angle
10. In fig (i) B is the "vertex" where the line segments BA and BC meet.
11. In fig (ii) E is the "vertex" of  $\angle DEF$  or  $\angle FED$ .

12. In fig. (i) line segments BA and BC are called as the "arms" or "sides" of the  $\angle B$  or  $\angle CBA$ .

13. In fig. (ii) the rays ED and EF are the "arms" or "sides" of the  $\angle E$  or FED.

Activities suggested to do in the classroom with the help of the students.

#### Activity I :

1 Ask the students to collect the following objects/solids – cuboid, cube, triangular prism.

2. With the help of a flat surface of the above solids, ask the students to draw the shapes on a sheet of paper to get the following figures – Rectangle, Square, Triangle.

3 Ask them to count the number of sides in Rectangle, Square and Triangle.

4 sides in a Rectangle, 4 sides in a Square, 3 sides in a Triangle

4 Ask them to list out the number of angles in each figure

[4 angles in a Rectangle, 4 angles in a square, 3 angles in a triangle].

5. Ask them to write down the number of vertices in each figure.

[ 4 vertices in a rectangle, 4 vertices in a square, 3 vertices in a triangle]

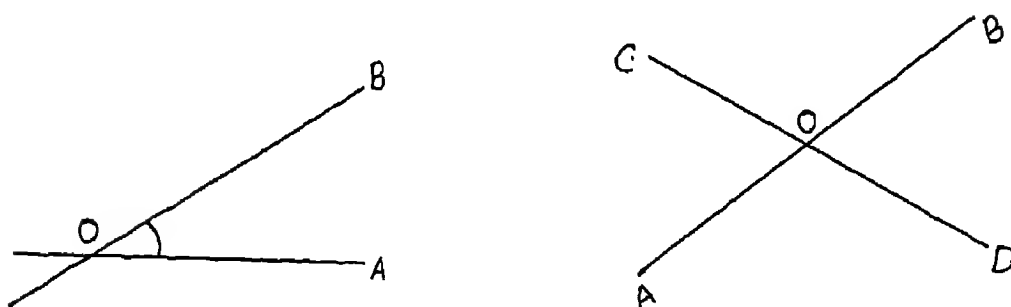
#### Activity II:

Ask the students to observe the following figures and answer the questions given below.

in fig. 1: a) What are the two sides of the angle? b) Which is the vertex of the angle? c) Write the name of the angle symbolically ?

In fig. 2 : The line segments AB and CD intersect at the point 'O'.

- a) How many angles are formed at 'O' ? b) Name all the angles formed at 'O' ? c) Which is the vertex of all these angles ?



Repeat the above activities by taking the objects like textbook, chalk box, pencil box, scale, forceps, tongs, cutting player, brick, tiles, etc. which are found in book stalls, shops, lab, hardware shops, medical stores, factories, etc.

### Exercises for Homework

- 1 Keep the door closed. Then open the door partially. See the angle formed by the horizontal edge of the door. Draw the figure of the angle so formed. Name it.

Repeat the above activity by using book, window, suit case and almirah.

- 2 Place your hands together with palms touching. Now keeping them touching at the elbows, gradually open the right hand outward. We say that you have turned your right hand through an angle. Here what happens when you turn your hand more? Does the angle increase?

Repeat the above activity by using a) two straws, b) two broom sticks, c) two pencils, d) two rod pieces.

3. Look at the corners of your book. Is a corner an angle? Ans. Yes

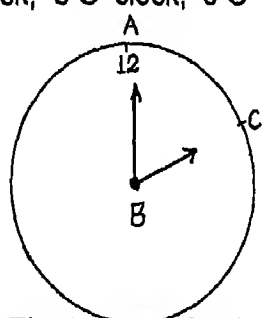
Can you find corners in the following objects ? Count the number of angles so found. a) Door plank, b) window shutter, c) a face of a rectangular rubber, d) rectangular sheet of paper

- a) Door plank 4 angles
- b) Window shutter 4 angles
- c) A flat surface of a rectangular rubber. 4 angles
- d) A sheet of paper 4 angles

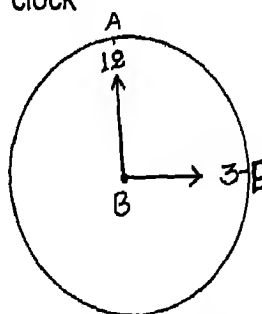
### 3.3.4 Sub-Unit : Identifies angles occurring in various shapes

**Introduction:** The teacher can ask the students to draw the angles formed in the following

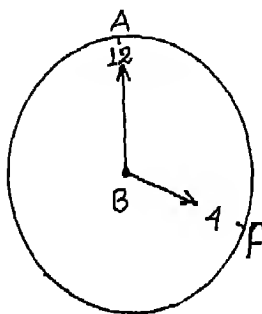
- 1 Look at the positions of minute hand and hour hand and draw the sketches of the angles so formed at 2 O' clock, 3 O' clock, 4 O' clock, 5 O' clock, 6 O' clock and 1 O' clock



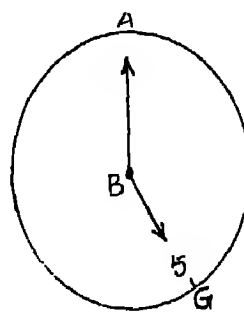
(a) The time is 2 O' clock



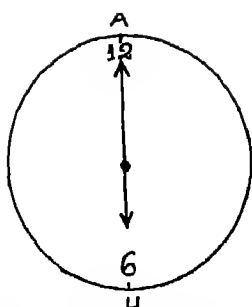
(b) The time is 3 O' clock



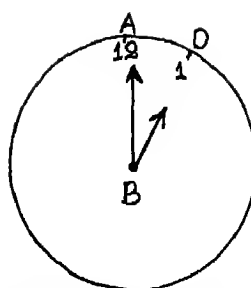
(c) The time is 4 O' clock



(d) The time is 5 O' clock.



(e) The time is 6 O' clock



(f) The time is 1 O' clock.

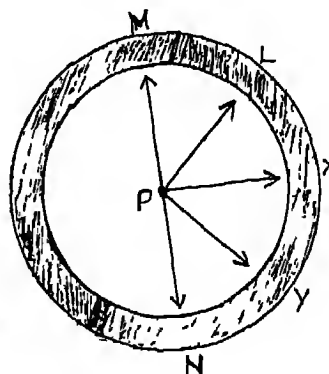
- Ans. a) ABC ,      b) ABE                      c) ABF  
             d) ABG      d) ABG                      e) ABD

2. Answer the following questions

- i) Compare ABC and ABE Which is greater ? [Ans ABE ]
- ii) Compare ABE and ABF. Which is greater ? Ans. ABF.
- iii) Compare ABF and ABG . Which is greater ? Ans . ABG
- iv) Compare ABG and ABH Which is greater ? ABH
- v) Compare ABC and ABD Which is greater ? Ans ABC
- vi) We can say easily
  - a)  $ABH > ABG > ABF > ABE > ABC > ABD$
  - b)  $ABD < ABC < ABE < ABF < ABG < ABH$

### Explanation /Activities

#### Activity 1:



Figure

Take a circular dial. Fix two sticks PM and PL at P as shown in the figure with the help of a pin. Rotate PL about P from M to N. Show the angle when L reaches X, Y and N

- 1 Draw the figures
- 2 Name the angles formed.
3. Does the angle increase as PL moves from PM to PN?
4. Which is the greatest angle?
- 5 Which is the smallest angle?



**Activity 2 :**

Draw the figure of a clock. Mark the following times on the dial.

- a) 3 h : 30 m      b) 8 h : 45 m      c) 6 h : 00 m  
d) 9 h : 15 m

Draw the angles so formed? Name them. What is the greatest angle? Which is the smallest angle?

**Activity 3 :**

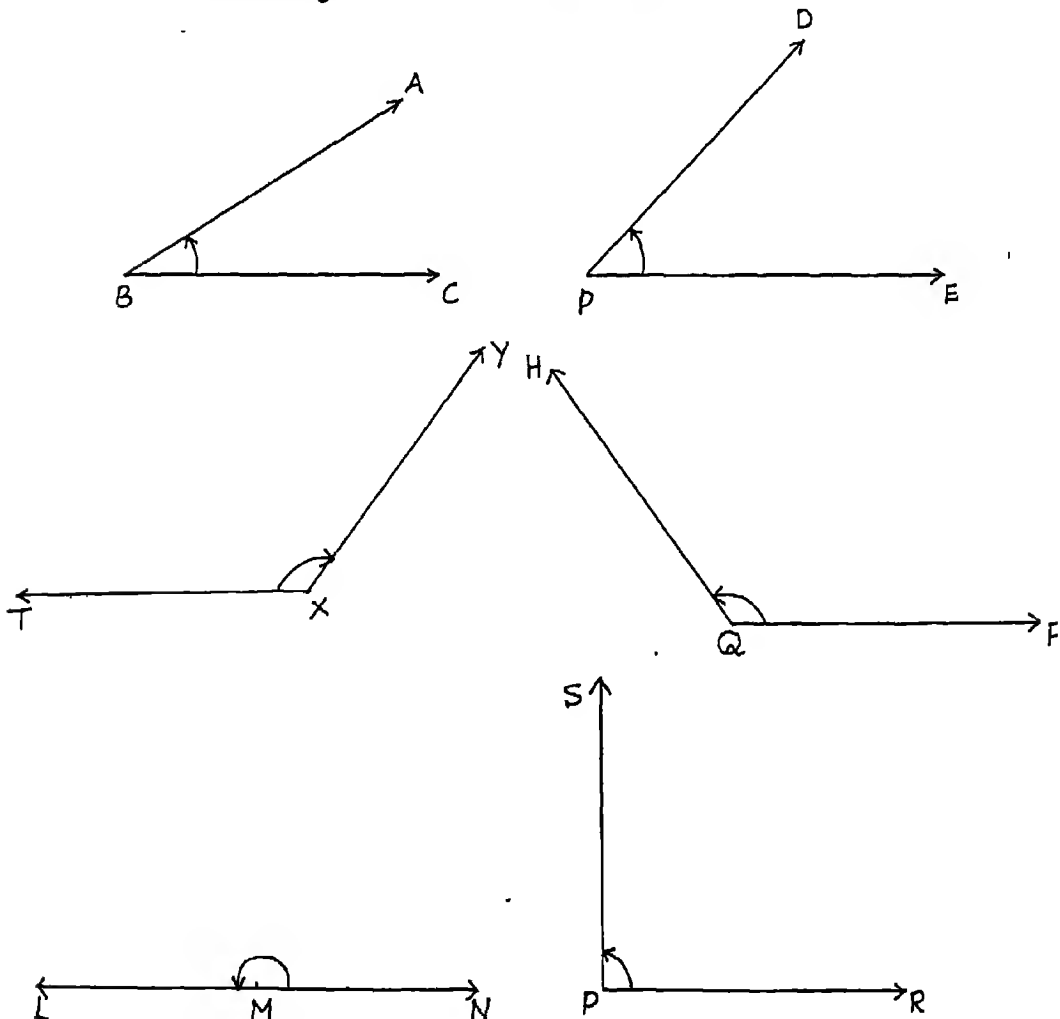
Draw the figures of the following English alphabets in capital letters?

- i) e; ii) f; iii) h; iv) t; v) l; vi) a; vii) m; viii) n; ix) w;  
x) y; xi) z xii) x xiii) v

Draw the angles so formed Name them Which is the greatest angle ?

Which is the smallest angle? Which are equal ?

In the following figures which of the angles ABC, DPE, HQF, LMN, TXY and SPR is the largest ? Which is the smallest ?



**Subunit No 3.4 Constructions and Geometrical facts:**

- 3.4-1 : Measuring and Drawing line segments
- 3.4-2 : Angles
- 3 4-3 : Construction of Circles.
- 3.4-4 : Geometrical Facts
- 3 4-4(i) :  $\text{Diameter} = 2(\text{Radius})$
- 3.4-4(ii) : Sum of the three angles of a Triangle
- 3 4-5 : Perpendicular Lines
- 3.4-6 : Parallel Lines.
- 3 4-7 : Certain Do's and Don'ts in drawing figures.

### 3.4 : CONSTRUCTIONS AND GEOMETRICAL FACTS

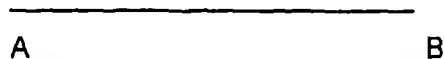
#### 3.4-1 : Measuring and Drawing line segments:

Here we will consider the uses of straight edged scale and divide, which are among the equipments of any mathematical instrument box. The picture of these equipments are given below for your easy recognition.

1)	Picture of a straight-edged scale
2)	Picture of a compass.

#### Activity 1:

Measure the length of the given line segment AB ( 8 cm in length)



Place the centimeter side of the straight edged scale along the line segment AB so that the initial point (0 marking point) of the scale falls on the point A. Now read the marking on the scale corresponding to the point B. Which is 8 cm in this case

#### Activity 2:

Construct a line segment of 7.5 cm



Press the scale evenly on a sheet of paper and note the points A & B on the paper against the markings "0" and "7.5" of the scale. Draw a line segment A and B. Now AB is a line segment of length 7.5 cm.

#### Activity 3:

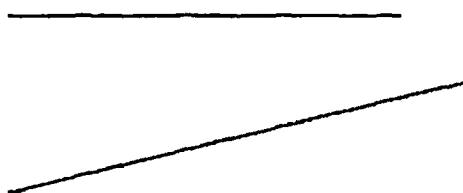
Draw a line segment whose length is that of the following line segments AB



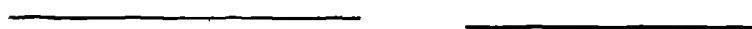
Draw a line segment longer than AB. Place the two end points of the divider on A & B. Now keeping the divider as it is, place two ends of the divider on the longer line segment, one leg evenly at L, other leg of the divider at a point on the line. Call this point as M. Now LM is the required line segment.

### Exercise

- 1) Measure the lengths of the following line segments: (using both scale and divider)

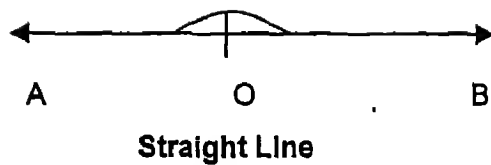


- 2) Draw line segments of length 4 cms, 5.2 cms and 6.8 cms
- 3) Draw line segments whose lengths are the lengths of the following line segments.



### 3.4-2: Measurement of Angle:

Angle formed by two opposite rays with common end point is called a straight angle. A straight angle is divided into 180 equal parts. Each part being called a **degree**. If each part is called a unit angle, then the measure of 1 unit angle is  $1^\circ$  (read 1 degree). Protractor is an instrument in the form of a semi-circle (see figure) used to measure an angle. The line segment joining  $0^\circ$  and  $180^\circ$  marks is known as the **base line** of the protractor. The mid point of the base line is known as the center of the protractor.



### How to measure an angle.

Place the center of the protractor on the vertex of the angle and adjust the protractor such that the base line is along one of the arms of the angle. Look in which direction of the base line this arm is, counting from the '0' in this direction in the clockwise/anticlockwise direction, read the mark on the protractor where the other arm of the angle crosses the semi circular scale of the protractor

Ex. Measure AOB in fig 1, and LOM in fig 2

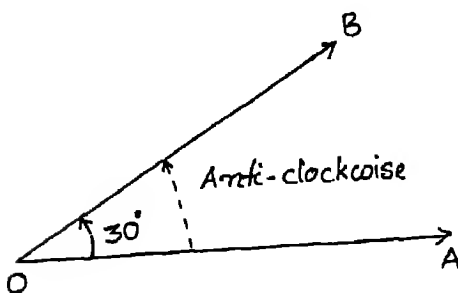


Fig.1

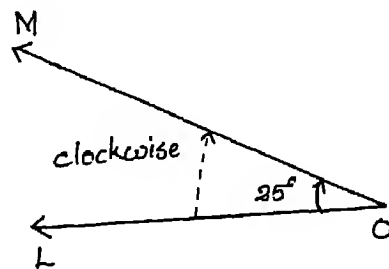
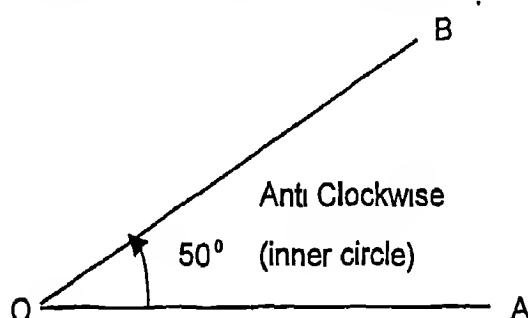


Fig.2

Following the procedure described in the last paragraph we have placed the center of the protractor on the point O in both diagrams and the base line along OA and OL in figure 1 and 2 respectively. In Figure 1 we measure the angle clockwise and in Figure 2 we measure the angle anticlockwise. The other ray of the angle intersects the outer semi circle of the protractor at C and N respectively in figure 1 and 2. Measuring the angles we find that angles AOB and LOM measure  $30^\circ$  and  $25^\circ$  respectively.

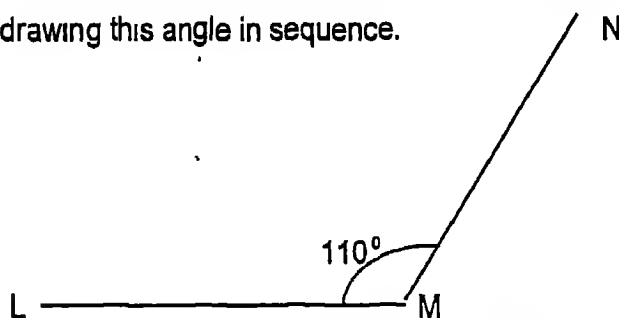
### How to construct an Angle

Now we shall describe how to construct angle of given measure (say  $50^\circ$ ). Draw a ray say OA, place the base line of the protractor along the ray OA with the center of the protractor on the point O. See whether it is the inner circle or outer circle whose  $0^\circ$  lies on the ray OA. Here the inner scale lies on OA. Mark the point B on the outer circle corresponding to  $50^\circ$  of the inner scale. Join OB. Now AOB is the required angle whose measure is  $50^\circ$ .



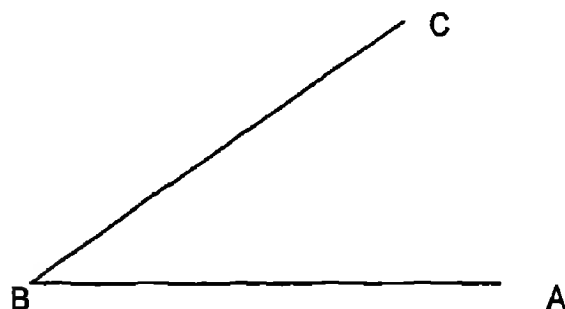
### Exercise for Self Evaluation:

- 1) The following LMN measures  $110^\circ$ . Describe the steps involved in drawing this angle in sequence.

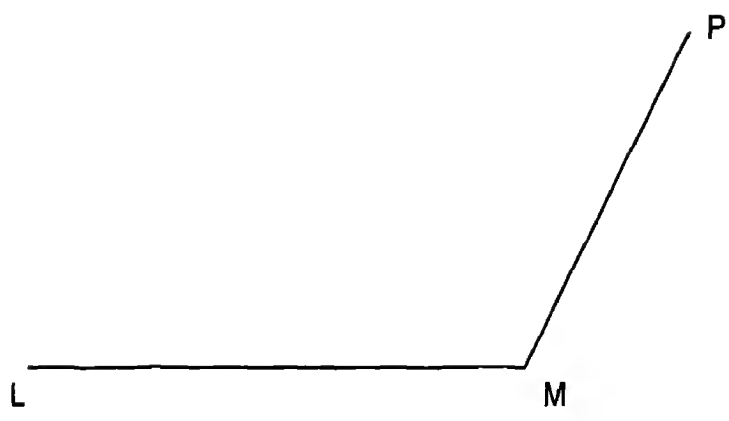


- 2) Measure the following angles and name them.

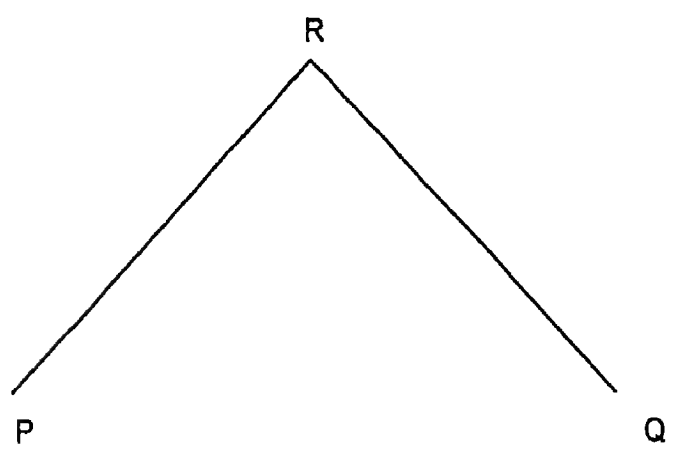
(i)



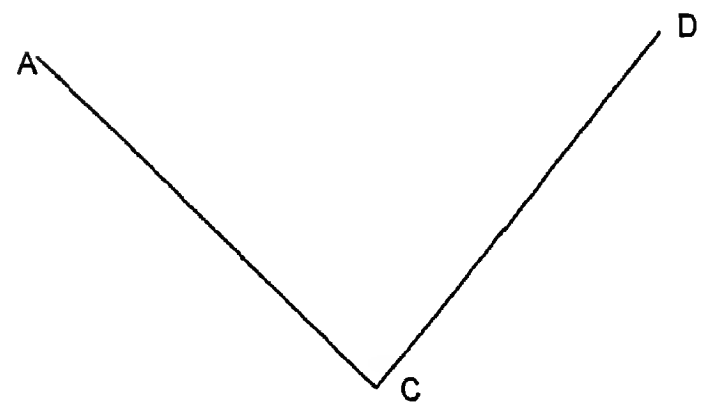
(ii)



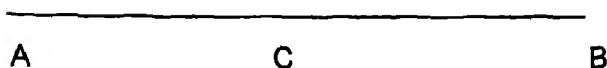
(iii)



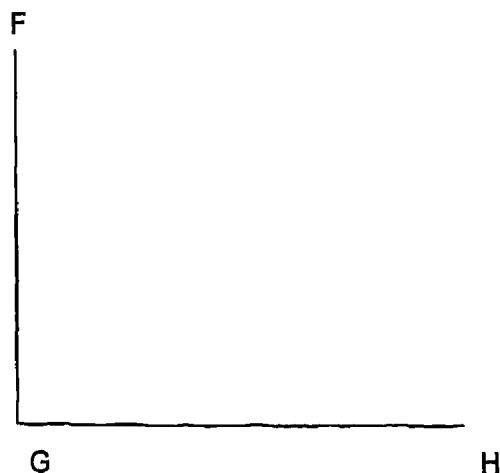
(iv)



(v)



(vi)



- 3) Using protractor construct the angles of measures  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $130^\circ$ , &  $150^\circ$ .

#### -Enrichment Exercises for Teachers.

- 1) With the help of set squares draw angle of measures  $15^\circ$  and multiples these of

#### Types of Angles

- 1) An angle whose measure is between  $0^\circ$  and  $90^\circ$  (excluding  $90^\circ$ ) is called an "acute angle".
- 2) An angle measuring  $90^\circ$  is called a "right angle".
- 3) An angle whose measure is between  $90^\circ$  and  $180^\circ$  (excluding  $90^\circ$  and  $180^\circ$ ) is called an "Obtuse angle".



4) An angle measuring  $180^\circ$  is called a "Straight angle".

For examples,

- (a) An angle measuring  $45^\circ$  is an acute angle.
- (b) An angle measuring  $130^\circ$  is called an Obtuse angle.

The relation among these types of angles may be stated as follows.

An acute angle < A right angle < An Obtuse angle < A straight angle

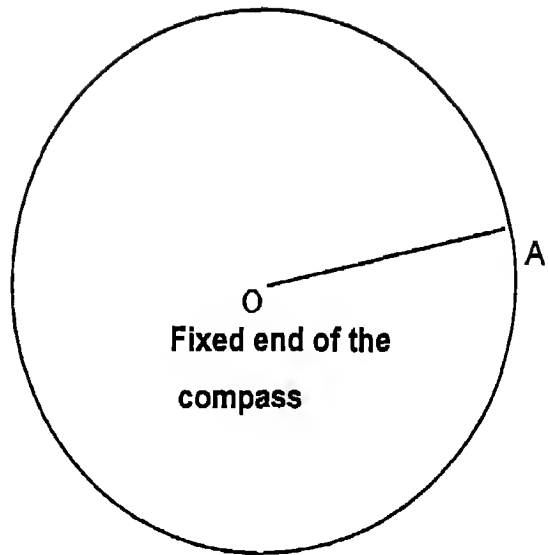
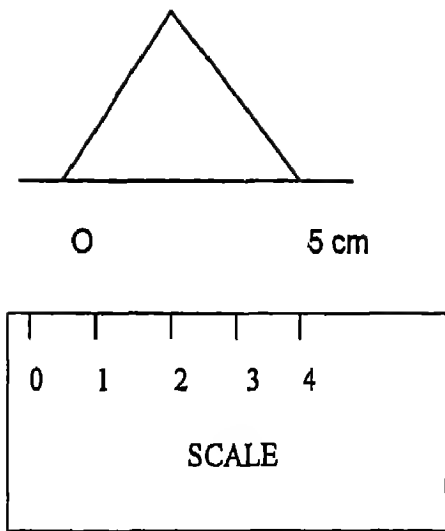
Fill in the blanks: (See page 56 and 57)

1)

Sl. No. of the fig.	Name of the angle	Type of the angle	Reason
2(i)	CBA	Acute Angle	Less than $90^\circ$
2(ii)	LMP		
2(iii)	PRQ		
2(iv)	ACD		
2(v)	ACB		
2(vi)	FGH		

3.4-3: Construction of circles:

Now we will describe how to draw a circle of radius 5 cm. Mark a point on a sheet of paper as the center of the circle. The point should be so located that it is possible to draw the circle. Place the sharp end of the compass at zero of the straight scale and move the other end of the compass with pencil point at 5 cm mark of the scale. Now keeping the end of the compass at O fixed, move the pencil point around to draw the desired circle.



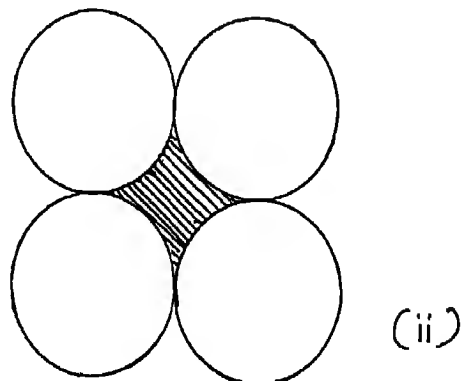
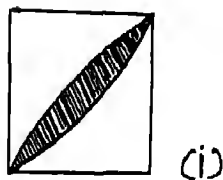
### Activities:

Draw circles with the following radii

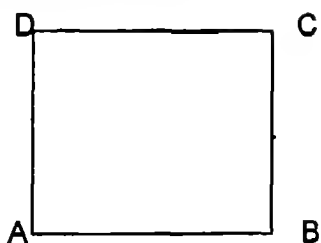
- (i) 6 cm (ii) 8.5 cm (iii) 7 cm (iv) 4.5 cms

### Further Activities:

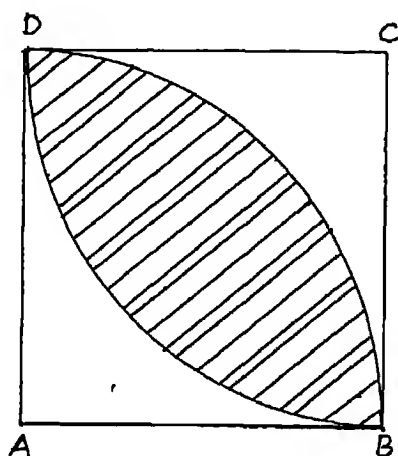
We can draw different patterns by drawing circles. Two examples are given below



(i) Draw a square ABCD

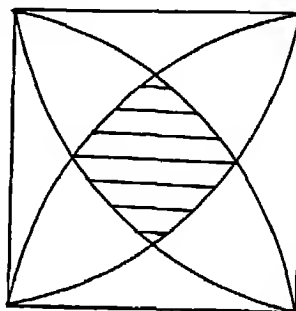
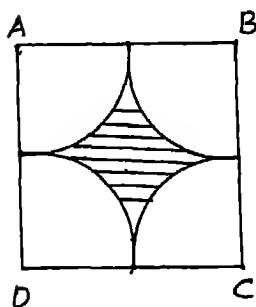


Draw quarter circles with A & C as center and the radius having the measures of the side, and shade their intersection



Now we have the first pattern.

(ii) Draw a square ABCD of a side of such a measure whose midpoint can easily be located. (say 6 cms) Draw circles with vertices with A,B,C and D as centers and each of the radii having the measure 3 cm. Shade the portion bounded by the four circular arcs (ie part of a circumference)



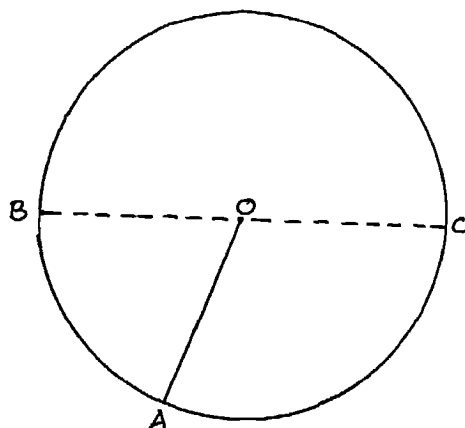
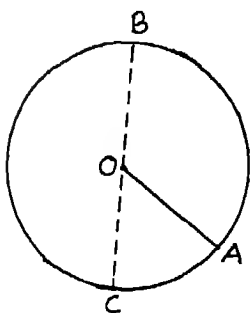
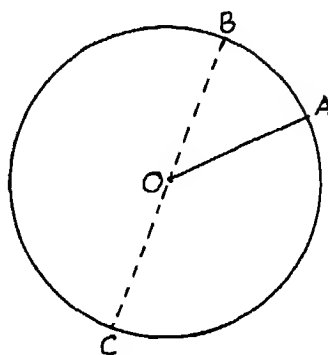
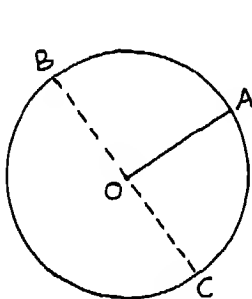
**Exercises for self evaluation:**

Draw the following pattern and write the steps involved in sequential order.

**3.4-4: GEOMETRICAL FACTS**

3.4-4(l) : Diameter = 2 (radius)

Draw as many circles as possible (say four) of different radii



Now fill in the following table.

	OA	BC	BC/OA=?	BC=——OA	Diametre = -----Radius)
Circle 1					
Circle 2					
Circle 3					
Circle 4					

After measuring, the students will conclude the  $\frac{BC}{OA} = 2$  or  $BC=2.OA$  or

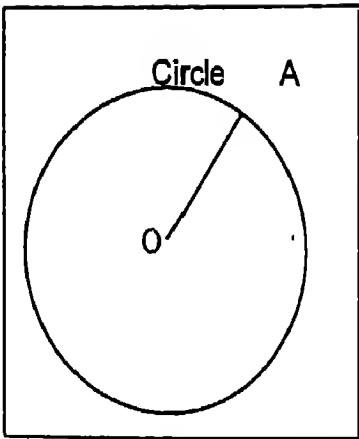
Diametre = 2 (Radius).

Circumference and diameter

3.4.4 (II)

Activity 1:

Draw a circle of a radius of known length on a thick card board and cut along the circumference. So a thick circular cut out is ready. Take a thread sufficiently long and fix one end of the thread at a fixed point of the circumference wrap the thread along the circumference till you comes back at the fixed point where you had started Measure the length of the thread. This is the circumference of the circle. Compare the lengths of diameter (twice the radius) with that of circumference. Repeat this experiment at least 3 or 4 times with different radii



Thick card board  
OA: is the radius of known length.  
O: is the center.

Form a table:

	Radius	Diam	Circum	$\frac{c}{d}$

Observation:  $\frac{c}{d}$  is a constant

We will find in each case that circumference is approximately 3 times the diameter

**Activity 2:**

Draw a sufficiently long straight line on a cardboard. Let the circular cardboard (of the activity) cut out roll without sliding along this line keeping a fixed point at one end of the line. When this fixed point is again on this line, then this cutout completes one complete revolution.

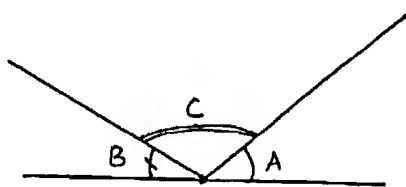
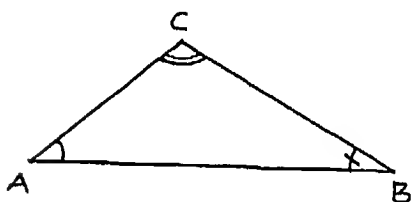
Traversing a length = circumference

Compare the length of radius and circumference

**3.4-4(III) : Sum of the Three Angles of a Triangle****Activity:**

Draw a triangle ABC on a piece of paper and cut apart the three angles A, B and C of the triangle by cutting along the sides. Then paste them together as shown below. You will find that they form a straight angle i.e the measure of  $A+B+C$  is equal to  $180^\circ$

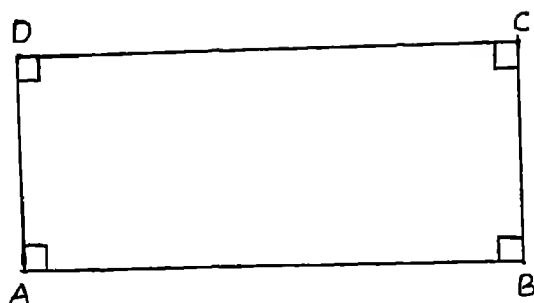
Repeat this experiment at least 3 or 4 times, drawing different triangles with different sizes.



So students conclude that 'Three angles of a triangle are together of measure  $180^\circ$

### 3.4-5 : Perpendicular Lines

#### Activity 1:

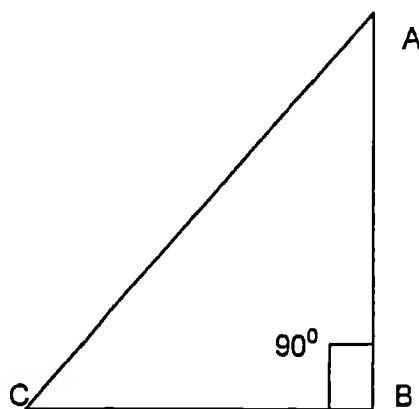


ABCD is a rectangular board. Here  $\hat{A}$  is a right angle. We also say "AB and AD are perpendicular lines" or "AB and AD are perpendicular to each other" or "AB is perpendicular to AD" Or "AD is perpendicular to AB".

The same thing can be said of  $\hat{B}$ ,  $\hat{C}$  and  $\hat{D}$ . For example, at the point B, BA and BC are perpendicular lines.

#### Activity 2:

Consider  $\triangle ABC$  which represents  
A triangular field or a triangular  
Flag



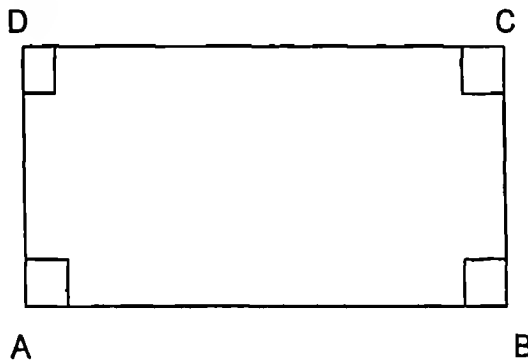
Here AB and BC are perpendicular lines, the measure of  $\hat{B} = 90^\circ$ . But the measure of  $\hat{A} \neq 90^\circ$ , so CA and AB are not perpendicular lines. For similar reason (why?) CA and AB are also not perpendicular lines.



Now we can say, that if an angle is a right angle, then its arms (or sides) are perpendicular lines

### 3.4-6 : Parallel Lines:

Ex 1. Consider the opposite edges of the top of a rectangular table or board ABCD.

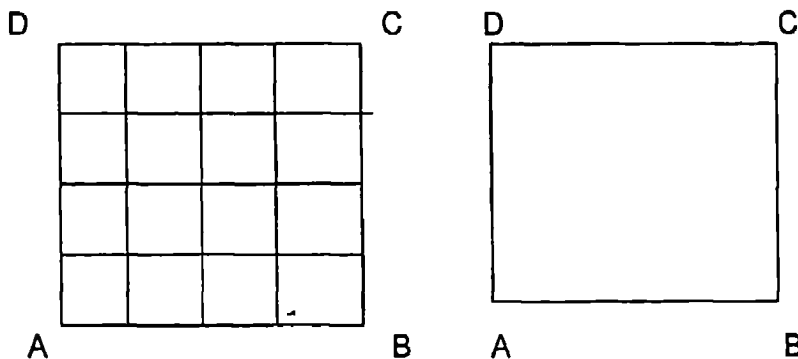


Here A, B, C, and D are all right angles. Also note that  $AD=BC$  and  $AB=CD$ . In such case we say that sides "AB and CD are parallel" or "sides AB and CD are parallel".

What can you say about sides AD & BC? Are they parallel?

The answer is "yes" why?

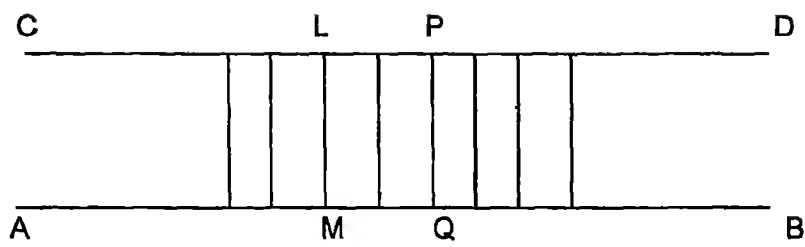
Ex 2.



Consider a square chessboard ABCD. What can you say about lines (i) AB and CD and (ii) AD and CB. Here each pair gives parallel lines.

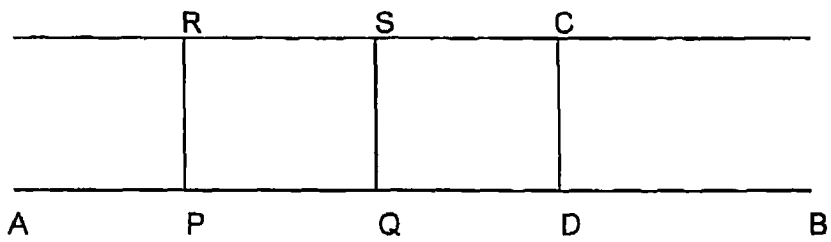
Find pairs of parallel line segments in the figure.

Ex-3:-



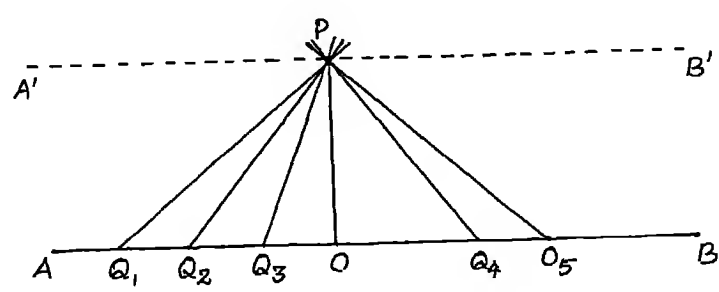
The above is a picture of railway track with wooden sleepers. Here at each point of a sleeper and any of the railway lines a right angle is formed. Also  $LM=PQ$ . So AB and CD are parallel lines  
What can you say about lines LM & PQ?

Ex-4:



Select two points P and Q on a straight line AB Draw lines PQ and QS perpendicular on AB such that  $PR=4\text{cm}$  and  $QS=4\text{cm}$  Draw the straight line through R and S Now RS and AB are parallel. Now select a point C on RS produced Draw the line CD perpendicular to RS so that D is on AB.  
Then what will be the length of CD? It will be of 4 cm  
(i) Now are lines RP and QS parallel? why?  
(ii) Now select any two horizontal lines Are they parallel? Why?

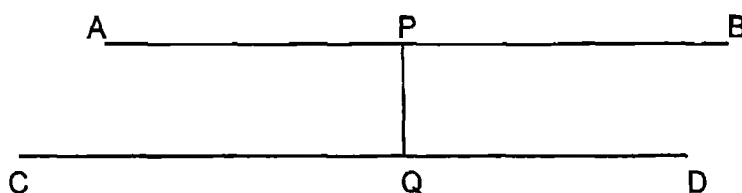
Ex-5:



Draw a line AB on a sheet of paper and choose a point P outside AB. Draw straight lines through P,  $PQ$ ,  $PQ_1$ ,  $PQ_2$ , and the (dotted) line  $A'PB'$  through A. Now observe that all straight lines except  $A'PB'$  intersect AB. Here  $A'PB'$  does not intersect AB and PO is perpendicular on AB. AB and  $A'B'$  are parallel.

**Def** Two straight lines are called **Parallel lines** if they do not intersect.

**Fact:**



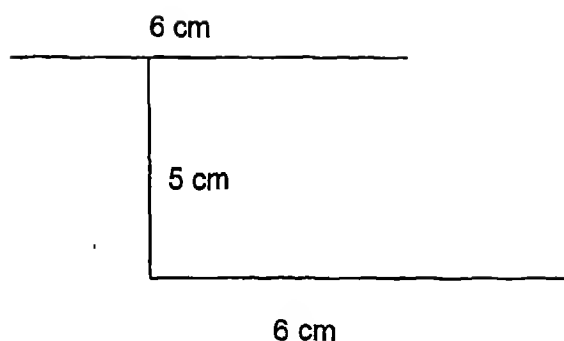
Let AB and CD be parallel lines and P be on AB. Draw the line PQ perpendicular on CD where Q is a point on CD. Then PQ is of the same length where ever the point P be on AB., PQ is called the distance between AB and CD.

#### Exercise for Self Evaluation:

- 1) Draw a straight-line segment and then draw a line segment of length 4.5 cm parallel to the first line segment
- 2) Draw two parallel line segments of length 5 cm at a distance of 4 cm from one another
- 3) Use set square to draw an angle of measure  $90^\circ$

#### Enrichment Activity:

Set Square is two right angular flat equipments with two graduated (are marked) sides. One triangle is with angle  $30^\circ$ ,  $60^\circ$ ,  $90^\circ$  and the other with  $45^\circ$ ,  $45^\circ$ ,  $90^\circ$ . Below is drawn a line segment of 6 cm. Use set squares to draw a line segment of the same length at a distance of 5 cm. Explain how you will place the set square to draw the construction.



### 3.4-7: Certain Do's and Don'ts in Drawing Constructions

There are certain Do's and Don'ts, which the teacher should strictly enforce to be followed by the students, while teaching construction. Some of them are given below.

The student should

- 1) Draw the figure neatly
- 2) Use a pencil with a pointed end.
- 3) Not to use ink or ballpoint pen
- 4) Handle the instruments care fully.
- 5) Leave all the traces of construction after the construction is over

Since the geometry is a practical subject at this level, the teacher should insist on students drawing the constructions in the class itself, besides those constructions, which they do as homework. This is very necessary because to get mastery in a skill-oriented subject, enough practice over a long period of time is the only way.

### Exercises for Self Evaluation

Give reasons for the following:

- 1) Pencil should be sharp to draw thin line in construction
- 2) After the construction is over, the student should leave the traces of construction in a class test.



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- 2 Shri B C Basti
- 3 Dr Anil Kumar
- 4 Dr Vijaya Raghavan

# **DEVELOPMENT OF SELF INSTRUCTIONAL MATERIALS IN MATHEMATICS FOR PRIMARY SCHOOL TEACHERS**

**Workshop for Development of Self Instructional Materials  
25.11.2003 – 4.12.2003**

## ***Resource Persons***

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2. Shri G Chandrashekar, RIMSE, Mysore
3. Dr V Shankaram, Mysore
4. Dr N B Badrinarayanan, Mysore
5. Shri G N G Dikshit, Registrar,  
M K Educational Trust, Harihar
6. Dr G Ravindra, Principal, RIE, Mysore
7. Dr N M Rao, RIE, Mysore
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# WORKSHOP TO REVIEW SELF-INSTRUCTIONAL MATERIALS IN MATHEMATICS

24 – 28 February 2003

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13. Ms C V Vijayalakshmi  
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15. Ms N Harini  
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